



The equivalence of two inequalities for quasisymmetric designs

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Abstract

It has been an open problem whether Hobart's inequality on the parameters of a quasisymmetric 2-design is independent of earlier known restrictions. In this note we show that it is equivalent to inequalities found by Neumaier and Calderbank. We also give some more parameter sets ruled out by the Blokhuis–Calderbank inequality.

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1 Quasisymmetric designs

A *design* is a finite set called the point set, provided with a collection of subsets called blocks. A t -(v, k, λ) *design* is a design with v points, where all blocks have size k and any t distinct points are in precisely λ blocks.

A *quasisymmetric design* with intersection numbers x, y , is a design where distinct blocks meet in either x or y points, where x, y are distinct and both occur [5,6].

A *strongly regular graph* with parameters (v, k, λ, μ) is a finite undirected graph without loops, having both edges and nonedges, with v vertices, regular of valency k , where two distinct adjacent (resp. nonadjacent) vertices have precisely λ (resp. μ) common neighbours. In this note we shall write (V, K, Λ, M) for the parameters of a strongly regular graph, to avoid a clash with design parameters.

Let (X, \mathcal{B}) be a quasisymmetric 2-(v, k, λ) design with intersection numbers x, y , where $1 < k < v$. The number of blocks on each point is $r = \lambda(v - 1)/(k - 1)$ and the total number of blocks is $b = vr/k$.

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