Exercise 1  Mutual Exclusion algorithm
a. Give the model for the algorithm given in Figure 1. in the lecture notes. Draw the graphical representation of the model for the algorithm given in Figure 1. in the lecture notes and compare it with the one from a. Does this algorithm ensure mutual exclusion and eventual access? Why?

b. Are there any states in the model from a. with the same valuation?

c. There are two states in the model from a., let’s say s₀ and s₀’, for which the valuation function gives V(s₀, n₁) = V(s₀', n₁) = tt and V(s₀, n₂) = V(s₀', n₂) = tt (assuming your model is correct). Do they have the same valuation? If not, can you change the algorithm so that s₀ and s₀’ have the same valuation?

d. Is it always possible to ”merge” two states with a same valuation into one state? Why?

Exercise 2  Consider a road-crossing as given in Figure 1 where A₁ and A₂ are traffic lights. Aᵢ can turn from red to red/yellow, then to green, then to yellow from which it turns back to red.

a. Give separate models for A₁ and A₂ and draw the graphical representation of the road-crossing traffic-light system.

b. Does your model guarantee the safety property that A₁ and A₂ are not both green at the same time?

c. Does your model satisfy the properties you would desire? If not, correct it.

Exercise 3  Consider again the mutual exclusion problem between two processes P₁ and P₂. An alternative solution is to model a binary semaphore Arbiter that regulates access to the critical section by a separate parallel process that interacts/synchronizes with P₁ and P₂. For simplicity, we take P₁ and P₂ to be modeled as given in Figure 2. Model the Arbiter that controls processes P₁ and P₂ in a way that the whole system satisfies the mutual exclusion property.

Exercise 4  In Modal Propositional Logic which of the following formulas are valid:

1. ⊨ □◊(p ∨ q) ⇒ □◊p ∨ □◊q
2. ⊨ □◊(p ∨ q) ⇐ □◊p ∨ □◊q
3. ⊨ ◊□(p ∨ q) ⇒ ◊□p ∨ ◊□q
4. ⊨ ◊□(p ∨ q) ⇐ ◊□p ∨ ◊□q
Exercise 5 Describe the meaning of the following LTL formulas in words:

1. \( \varphi \Rightarrow \Diamond \psi \)
2. \( \Box (\varphi \Rightarrow \Diamond \psi) \)
3. \( \Diamond \Box \varphi \)
4. \( \Diamond \Diamond \varphi \)
5. \( \Box (\varphi \Rightarrow \Box \varphi) \)

Exercise 6 Consider the formulas from Exercise 4, now interpreted as LTL formulas. What can you say about their validity?

1. \( \models \Box \Diamond (p \lor q) \Rightarrow \Box \Diamond p \lor \Box \Diamond q \)
2. \( \models \Box \Diamond (p \lor q) \Leftrightarrow \Box \Diamond p \lor \Box \Diamond q \)
3. \( \models \Diamond \Box (p \lor q) \Rightarrow \Diamond \Box p \lor \Diamond \Box q \)
4. \( \models \Diamond \Box (p \lor q) \Leftrightarrow \Diamond \Box p \lor \Diamond \Box q \)

Exercise 7 Argue whether the following equivalences of LTL formulae are valid or not:

1. \( \Box \varphi \land \Box \psi \Leftrightarrow \Box (\varphi \land \psi) \)
2. \( \Diamond \varphi \land \Diamond \psi \Leftrightarrow \Diamond (\varphi \land \psi) \)
3. \( \varphi \lor \psi \Leftrightarrow \varphi \lor (\varphi \lor \psi) \)
4. \( \Box (\varphi \Rightarrow \Diamond \psi) \Leftrightarrow (\varphi \lor (\varphi \Rightarrow \psi)) \)
5. \( \Diamond (\varphi \lor \psi) \Leftrightarrow \Diamond \psi \)

Exercise 8 Let model \( M \) be given as a sequence of states with \( PV = \{p, q, r, s, t\} \)

Interpret the LTL formulas \( \Box p, \Diamond t, \Box \Diamond s, p \lor q, \Box (r \Rightarrow (q \lor s)) \) on each (path starting in) state of \( M \).

Exercise 9 The goal of this exercise is to specify some properties of an elevator system. Assume that there is an elevator door at each floor of the building with an "up" and a "down" button, and one button for each floor in the elevator cabin.

Use the following atomic state properties (propositions) in your specification:

- \( at_i \): The elevator is at the \( i \)th floor
- \( open \): The elevator door is open
- \( open_i \): The door at the \( i \)th floor is open
- \( press_i \): Someone is pressing the button for the \( i \)th floor inside the elevator
- \( press_{up_i} \): Someone is pressing the "up" button on the \( i \)th floor
- \( press_{down_i} \): Someone is pressing the "down" button on the \( i \)th floor

Describe the following properties by PLTL formulae:

a. The elevator is never at the first and second floor at the same time
b. If a button is pushed on some floor, the elevator will serve that floor
c. A floor door is only open if the elevator is at that floor
d. Again and again the elevator returns to the \( i \)th floor
e. If no button is pushed and the elevator is at the \( i \)th floor, it will wait at that floor until a button is pushed.