Example 1 Consider the program $P$ given below:

```plaintext
out z : integer
in x, y : integer
z = min(x, y)
l_0 : if (x > y) {
    l_1 : z = y;
} else {
    l_2 : z = x
} l_3 :}
```

Show that $p : at l_3 \rightarrow z = \text{min}(x, y)$ is $P$-invariant.

**Solution** The invariant we want to prove is not strong enough, thus it must be strengthened. We try a stronger formula (and try to prove it as a $P$-invariant)

$\varphi : (at l_0 \rightarrow z = \text{min}(x, y)) \land (at l_1 \rightarrow x > y) \land (at l_2 \rightarrow x \leq y) \land (at l_3 \rightarrow z = \text{min}(x, y))$

Clearly, $\Theta \rightarrow p$ as $\Theta : z = \text{min}(x, y)$ and $\varphi \rightarrow p$.

Next we have to show that for every transition $\tau$ it holds: $\varphi \tau \varphi$.

For instance for $\rho_{\text{move}}^{l_0} : \text{move}(l_0, l_1) \land x > y \land x' = x \land y' = y \land \varphi \rightarrow \varphi'$ which after simplification becomes:

$\text{move}(l_0, l_1) \land x > y \land x' = x \land y' = y \land (at l_0 \rightarrow z = \text{min}(x, y)) \rightarrow (at l_1' \rightarrow x' > y')$ which obviously holds.

In a similar way we check for all transitions and conclude that $\phi$ is invariant. Therefore, $p$ is a $P$-invariant.