Exercise 1 Explain why the following formulas are not well-formed CTL formulas:

1. $FGq$
2. $XXr$
3. $A\neg G\neg p$
4. $F[r \cup q]$
5. $EXXr$
6. $AEFr$
7. $AF[(r \cup q) \land (p \cup r)]$

Exercise 2 Consider the model $\mathcal{M}$ in the figure below. Check whether $\mathcal{M}, s_0 \models \varphi$ and $\mathcal{M}, s_2 \models \varphi$ for the CTL formula $\varphi$:

1. $AFq$
2. $AG(EF(p \lor r))$
3. $EX(EXr)$
4. $AG(AFq)$

Solution

1. $s_0 \models AFq$, $s_2 \models AFq$, $s_3 \models AFq$. $s_1 \not\models AFq$ as path $s_1s_1s_\ldots \not\models Fq$
2. $s \models AG(EF(p \lor r))$ for any state in the model
3. $s \models EX(EXr)$ for any state in the model
4. $s \not\models AG(AFq)$ for any state $s \in \{s_0, s_1, s_2, s_3\}$, as for any $s$ there is a path that passes through $s_1$ and $s_1 \not\models AFq$.

Exercise 3 Which of the following pairs of CTL formulas are equivalent? For those which are not, find a model of one of the pair which is not a model of the other:

1. $EF\varphi$ and $EG\varphi$
2. $EF\varphi \lor EF\psi$ and $EF(\varphi \lor \psi)$
3. $AF\varphi \lor AF\psi$ and $AF(\varphi \lor \psi)$
4. $AF\neg \varphi$ and $\neg EG\varphi$
5. $EF\lnot\varphi$ and $\lnot AF\varphi$
6. $true$ and $AG\varphi \rightarrow EG\varphi$
7. $true$ and $EG\varphi \rightarrow AG\varphi$

Exercise 4 Find a model which distinguishes the following pairs of CTL* formulas:

1. $AFGp$ and $AFAGp$
2. $AGFp$ and $AGEFp$
3. $A[(p \cup r) \lor (q \cup r)]$ and $A[(p \lor q) \cup r]$
4. $EGFp$ and $EGEFp$

Exercise 5 Express the following properties in CTL and LTL whenever possible. If neither is possible, try to express the property in CTL*

1. Possibly the system never goes down;
2. Invariantly the system never goes down;
3. It is always possible that the system returns to the initial state
4. The system always eventually goes down and is operational until going down

Exercise 6 Give example of an LTL-formula for which equivalent translation in CTL does not exist.

Give example of an CTL-formula for which equivalent translation in LTL does not exist.

Give example of an CTL* -formula for which equivalent translation in LTL either in CTL does not exist.

Solution
LTL not CTL: $AFGp$
CTL not LTL: $AFAGp$

Exercise 7 Let $\varphi$ be CTL* state formula. What is the relation between $\varphi$ and $E\varphi$? Also, what is the relation between $\varphi$ and $A\varphi$?

Solution It is important that $\varphi$ is a CTL* state formula. From the CTL* semantics it follows immediately that $\varphi \leftrightarrow E\varphi$ and $\varphi \leftrightarrow A\varphi$. 

Solution LTL not CTL: $AFGp$

Solution CTL not LTL: $AFAGp$

Solution CTL* not LTL neither CTL: $AFGp \land AF AG p$