Exercise 1 Using System G prove that the following sentences is a tautologies:

\[(\exists x A \rightarrow \forall x B) \rightarrow \forall x (A \rightarrow B)\]

<table>
<thead>
<tr>
<th>Figure 1: The parallel system consists of two components.</th>
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<tr>
<td><img src="#" alt="Parallel System Diagram" /></td>
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Exercise 2 Consider the computations of the system in the figure above.

i. If all transitions are just do you have that \( t_2 \Rightarrow \diamond c_2 \)?

ii. If all transitions \( \tau_{\{t_1, t_2\}, \{c_1, c_2\}} \) are made compassionate do you have \( t_2 \Rightarrow \diamond c_2 \)?

iii. If again all transitions, except \( \tau_{\{c_1, m\}, \{c_1, m\}} \), are just, what do you minimally need to require for transitions \( \tau_{\{c_1, m\}, \{c_1, m\}} \) to achieve \( c_1 \Rightarrow \diamond c_1 \)?

**Solution** Hint: use the definition of just and compassionate transitions.

Exercise 3 Consider the system in the figure above once again with justice for all transitions. Prove:

i. \( \Box \neg (c_{1,1} \land c_{1,2}) \)

ii. \( \Box \neg (c_{1,1} \land c_{2}) \)

Transitions of the system are defined (in the standard way) by transition relations \( \rho \). For instance, transition relation for transition \( \tau_{\{t_1, t_2\}, \{c_1, t_2\}} \) is defined by \( \text{move}(\{t_1, t_2\}, \{c_1, t_2\}) \land \neg c_2 \). The initial condition of the system is \( \Theta : \pi = \{t_1, t_2\} \).

**Solution** Hint: Strengthen the formula \( \neg (c_{1,1} \land c_2) \) into a (potential) program invariant \( \varphi \). Show that for every transition \( \tau \) it holds \( \varphi \tau \varphi \).

For instance, for the transition of the overall system when the left component moves from \( t_1 \) to \( c_{1,1} \) and the right component does stay in \( t_2 \) we must show that:

\( \text{move}(\{t_1, t_2\}, \{c_{1,1}, t_2\}) \land \neg c_2 \land \varphi \rightarrow \varphi' \).

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1These are transitions of the cooperation when the right component is making a transition and the left component remains in the same state.