Exercise 1  Consider the CTL* formula over propositions \( P = \{a, b\} \): a.) \( f = AGF E(a U EG b) \) 
b.) \( g = AGF EX(a U EG b) \) and the transition system below:

Apply the CTL* model checking algorithm to compute all states for which the given formula is satisfied. (For LTL sub-formulas you may infer validity directly without applying the LTL model checking algorithm).

Solution States subformulas per levels:
level 0: \( a, b \)
level 1: \( EG b \) and lets denote \( p \equiv EG b \)
level 3: \( f \equiv AFGEq \) but we rewrite it as \( f \equiv \neg EG \neg q \).

Using CTL and LTL algorithms we can conclude that \( s_4, s_6, s_7 \vdash p \) and \( s_3, s_4, s_6, s_7 \vdash q \) from where \( s_0, s_1, s_3, s_4, s_6, s_7 \vdash f \).

Exercise 2  Consider the CTL* formula over propositions \( P = \{a, b\} \): \( f = E (X(a \land \neg b) \land X A (b U G a)) \).

Apply the CTL* model checking algorithm to compute all states of the transition systems given below, for which the given formula is satisfied. (For LTL sub-formulas you may infer validity directly).

Exercise 3  Consider the CTL* formula over propositions \( P = \{a, b, c\} \): \( f = EG F b \land G E F X a \land AGF X c \).

Apply the CTL* model checking algorithm to compute all states of the transition systems given below, for which the given formula is satisfied. (For LTL sub-formulas you may infer validity directly).

Solution For the formula \( f \equiv E(GF b \land GE F X a) \land AGF \neg(X c) \) we find the following levels of the state subformulas:
level 0: \( a, b \)
Consider the formula: $\mu Y.f \equiv (f \lor \Box Y)$ and $\nu Y.f \equiv (f \lor \Box Z)$. Reason about the paths that they determine. Now, consider the formula: $\mu Y.\nu Z.(p \land \Box Y) \lor \neg(p \land \Box Z)$.

1. Using your understanding of the previous two formulas reason about the third $\mu$-calculus formula and the paths it determines.

2. Apply the $\mu$-calculus model checking algorithm to compute the states of the transition system given below, for which the given formula is satisfied.

![Transition System Diagram]

**Solution** The given formula has nested fix point operators. Therefore, for every iteration of $Y_i$ a formula $\nu Z.(p \land \Box Y_i) \lor \neg(p \land \Box Z)$ should be evaluated and this gives $Y_{i+1}$.

$Y_0 = \emptyset$

Evaluate $Y_1 = \nu Z.(p \land \Box Y_0) \lor \neg(p \land \Box Z) = \nu Z.(p \land \Box \emptyset) \lor \neg(p \land \Box Z) = \nu Z.\neg p \land \Box Z$

$Z_{00} = \{1, \ldots, 6\}$

$Z_{01} = \{2, 3, 5\} \cap \{1, \ldots, 6\} = \{2, 3, 5\}$

$Z_{02} = \{2, 3, 5\} \cap \{2, 3, 4, 6\} = \{2, 3\}$

$Z_{02} = \{2, 3, 5\} \cap \{2, 3, 4\} = \{2, 3\} \rightarrow$ fix point

$Y_1 = \{2, 3\}$

Evaluate $Y_2 = \nu Z.(p \land \Box Y_1) \lor \neg(p \land \Box Z) = \nu Z.(p \land \Box \{2, 3\}) \lor \neg(p \land \Box Z)$

$Z_{10} = \{1, \ldots, 6\}$

$Z_{11} = \{4\} \cup \{2, 3, 5\} = \{2, 3, 4, 5\}$

$Z_{12} = \{4\} \cup \{2, 3, 5\} = \{2, 3, 4, 5\} \rightarrow$ fix point

$Y_2 = \{2, 3, 4, 5\}$

Evaluate $Y_3 = \nu Z.(p \land \Box Y_2) \lor \neg(p \land \Box Z) = \nu Z.(p \land \Box \{2, 3, 4, 5\}) \lor \neg(p \land \Box Z)$

$Z_{20} = \{1, \ldots, 6\}$

$Z_{21} = \{4, 6\} \cup \{2, 3, 5\} = \{2, 3, 4, 5, 6\}$

$Z_{22} = \{4, 6\} \cup \{2, 3, 5\} = \{2, 3, 4, 5, 6\} \rightarrow$ fix point

$Y_3 = \{2, 3, 4, 5, 6\}$
Evaluate $Y_4 = \nu Z.(p \land \Box Y_3) \lor (\neg p \land \Box Z) = \nu Z.(p \land \Box \{2, 3, 4, 5, 6\}) \lor (\neg p \land \Box Z)$

$Z_{30} = \{1, \ldots, 6\}$
$Z_{31} = \{2, 3, 4, 5, 6\} \cup \{2, 3, 5\} = \{2, 3, 4, 5, 6\}$
$Z_{32} = \{2, 3, 4, 5, 6\} \cup \{2, 3, 5\} = \{2, 3, 4, 5, 6\}$ $\rightarrow$ fix point

$Y_4 = \{2, 3, 4, 5, 6\}$ $\rightarrow$ fix point

Thus, $f$ is satisfies in states: 2,3,4,5,6.