Exercise 1 (Modal Propositional Logic)

Consider the model \( \mathcal{M} \) of the ferryman problem with two stones with \( PV = \{ f, s_1, s_2 \} \) and starting state \( p_1 \) with \( f \land s_1 \land s_2 \).

a. Check whether \( \mathcal{M}, p_1 \models \Box \neg f \) and \( \mathcal{M} \models f \Rightarrow \Box \neg f \)

b. Find all states (worlds) \( p \) such that \( \mathcal{M}, p \models \varphi \) where \( \varphi \) is:
   1. \( f \land (s_1 \lor s_2) \Rightarrow \Diamond (\neg s_1 \lor \neg s_2) \)
   2. \( f \land (s_1 \lor s_2) \Rightarrow \Box (\neg s_1 \lor \neg s_2) \)
   3. \( \Box (\neg s_1 \lor \neg s_2) \)
   4. \( \Diamond (\neg s_1 \lor \neg s_2) \)
   5. \( \Diamond (\neg s_1 \land \neg s_2) \)

Exercise 2 (PLTL) Let model \( \mathcal{M} \) be given as a sequence of states with \( PV = \{ p, q, r, s, t \} \)

Interpret the LTL formulas \( \Box p \), \( \Diamond t \), \( \Box \Diamond s \), \( q \lor s \), \( \Diamond (r \Rightarrow (q \lor s)) \) on each (path starting in) state of \( \mathcal{M} \).

Exercise 3 The goal of this exercise is to specify some properties of an elevator system.

Assume that there is an elevator door at each floor of the building with an "up" and a "down" button, and one button for each floor in the elevator cabin.

Use the following atomic state properties in your specification:

- \( at_i \) The elevator is at the \( i \)th floor
- \( open \) The elevator door is open
- \( open_i \) The door at the \( i \)th floor is open
- \( press_i \) Someone is pressing the button for the \( i \)th floor inside the elevator
- \( press_{up_i} \) Someone is pressing the "up" button on the \( i \)th floor
- \( press_{down_i} \) Someone is pressing the "down" button on the \( i \)th floor

Describe the following properties by PLTL formulae:

a. The elevator is never at the first and second floor at the same time
b. If a button is pushed on some floor, the elevator will serve that floor
c. A floor door is only open if the elevator is at that floor
d. Again and again the elevator returns to the \( i \)th floor
If no button is pushed and the elevator is at the \( i \)th floor, it will wait at that floor until a button is pushed.

**Exercise 4** Describe the meaning of the following LTL formulas in words, when interpreted on model \( M = \langle \pi, V \rangle \) where \( \pi = w_0w_1 \ldots : \)

1. \( \varphi \Rightarrow \Diamond \psi \)
2. \( \Box(\varphi \Rightarrow \Diamond \psi) \)
3. \( \Box \Diamond \varphi \)
4. \( \Diamond \Box \varphi \)
5. \( \Box(\varphi \Rightarrow \Box \varphi) \)

**Exercise 5** Prove or disprove the following equivalences of LTL formulae

1. \( \Box \varphi \land \Box \psi \equiv \Box(\varphi \land \psi) \)
2. \( \Diamond \varphi \land \Diamond \psi \equiv \Diamond(\varphi \land \psi) \)
3. If \( \varphi \lor \psi \) holds then \( \varphi \lor \psi \) also holds.
4. \( \Box \varphi \Rightarrow \Diamond \psi \equiv \varphi \lor (\varphi \Rightarrow \psi) \)
5. \( \Diamond(\varphi \lor \psi) \equiv \Diamond \psi \)

**Exercise 6** (CTL* and CTL)

Consider the model \( M \) in the figure below. Check whether \( M, s_0 \models \varphi \) and \( M, s_2 \models \varphi \) for the CTL formula \( \varphi \): 

1. \( AFq \)
2. \( AG(EF(p \lor r)) \)
3. \( EX(EXr) \)
4. \( AG(AFq) \)

**Exercise 7** Express the following properties in CTL and LTL whenever possible. If neither is possible, try to express the property in CTL*.

1. Whenever \( p \) is followed by \( q \) (after finitely many steps), then the system enters and ‘interval’ in which no \( r \) occurs until \( t \)
2. After \( p, q \) is never true.
3. There is a path on which \( p \) is true infinitely often.
4. Property \( p \) is true for every second state along a path.