

A. Isebree Moens and D.J. Korteweg: on the speed of propagation of waves in elastic tubes

A S Tijsseling
Department of Mathematics and Computer Science
Eindhoven University of Technology, The Netherlands

A Anderson
School of Mechanical and Systems Engineering
University of Newcastle upon Tyne, United Kingdom

ABSTRACT

The Moens-Korteweg formula for the speed of propagation of pressure waves dates back to 1878 and was used by Kries in haemodynamics and Frizell, Joukowsky, Allievi and others in waterhammer to calculate the pressure variations in unsteady pipe flows. This paper describes the life and work of Dutchmen Isebree Moens and Korteweg. Their doctoral dissertations (in Dutch) are partly translated, reviewed and compared with their key publications (in German). Korteweg gave Moens' semi-empirical wavespeed a mathematical basis and he made the first contributions toward the study of fluid-structure interaction and unsteady friction. Their work is placed in historical context, in terms of both their predecessors and contemporaries, and also how it was subsequently built on by their successors in different disciplines.

Key words

Pressure surges; Water hammer; Hydraulic transients; Haemodynamics; Wavespeed; History.

INTRODUCTION

The historical development of finding the correct formula for the speed of propagation of waves in elastic tubes is reviewed. Breakthroughs came in a very short period of time via the contributions of Rézal (1876) [1, 2], Moens (1877, 1878) [3, 4] and Korteweg (1878) [5, 6]. The emphasis in this paper is on Moens' wavespeed formula:

$$c_{\text{Moens}} = \alpha \sqrt{\frac{E e}{\rho_f D}},$$

which is valid for the propagation of disturbances in elastic cylindrical rubber tubes, and on Korteweg's extension:

$$\frac{1}{c_f^2} = \frac{1}{c_0^2} + \frac{1}{c_1^2} \quad \text{or} \quad c_f = \frac{c_0 c_1}{\sqrt{c_0^2 + c_1^2}} \quad \text{or} \quad c_f = \frac{c_0}{\sqrt{1 + \frac{K D}{E e}}} = c_{\text{Korteweg}},$$

which is valid for waterhammer (slightly compressible fluid in elastic distensible pipes). Korteweg combined the two speeds:

$$c_0 = \sqrt{\frac{K}{\rho_f}}$$

of waves in unconfined liquid (about 1480 m/s for water at room temperature) and Résal's formula:

$$c_1 = \sqrt{\frac{E e}{\rho_f D}} = c_{\text{Resal}} .$$

The mysterious factor α in Moens' formula will be examined herein. Its value is about 0.9 and it is somewhat related to the axial-constraint coefficient ψ used in classical waterhammer theory [7]:

$$c_{1\psi} = \sqrt{\frac{1}{\psi} \frac{E e}{\rho_f D}} \quad \text{with } \psi = 1, \psi = 1 - \nu^2 \text{ or } \psi = 1 - \frac{\nu}{2}$$

depending on pipe constraint. Herein we use the "modern" notation c for wave celerity, D for pipe diameter, e for pipe wall thickness ("épaisseur" in French), E for Young's modulus of elasticity, K for bulk modulus, ρ_f for fluid density and ν for Poisson's contraction coefficient.

The Dutch dissertations of Moens [3] and Korteweg [5] are summarised. Both of them are a pleasure to read, because of their clarity and ingenuity. From them it is evident that Moens is the medical engineer and Korteweg the applied mathematician. Moens' career went from engineering to physiology to medicine (see the Appendix for a short biography of both Moens and Korteweg). He performed an engineering analysis (mass-spring approach) and validated his findings by extensive and precise measurements. His publication in German [4] is a long version of his PhD thesis. Dow [8] gave an excellent description of Moens' life and work, and he discussed Moens' less well received papers [9, 10]. Korteweg's career as an engineer to be (studying in Delft) was not a success and he developed himself as a mathematician. His PhD study was supervised by no one less than the physicist Van der Waals and it shows his early interest in wave phenomena. His well known German paper [6] is a short version of his thesis. The "Stellingen" (Theses) showing the wider interests of Dutch doctoral candidates concern Medicine (Moens) and Physics (Korteweg).

THE 19TH CENTURY BACKGROUND TO THE CONTRIBUTIONS OF MOENS AND KORTEWEG

Contemporaneously with Thomas Young [11, 12] at the end of the 18th Century, Chladni conducted pioneer investigations on vibrations, including those of organ tubes, and published his treatise on sound "Die Akustik" in 1802 [13] (see, e.g., Miller [14], pp. 41-42; Wolf [15], pp. 174-176; Lambossy [16]). One of his discoveries was to prove to be of great importance in terms of driving subsequent researches into the speed of sound in tubes. The wavelength λ of the fundamental of an organ tube is double its length L , so that determining the frequency f gives the velocity of sound c in the tube by $c = f\lambda = 2fL$. Chladni, however, compared various fluids in the tubes and discovered in the experimental values inexplicable (to him) differences from the theoretical ratio $c_1/c_2 = \sqrt{(\rho_2/\rho_1)}$ (where ρ_f is fluid density). This problem was to give rise to a whole series of researches during the 19th Century. Savart (1825) [17] (see also Tokaty [18], p. 139), investigating the speed of sound for air in various tubes, discovered there was a change in frequency if more flexible walls were used. Cagniard (de) Latour (1834) [19] used

water in the tubes and found that this amplified the Savart effect in a fashion similar to that observed by Chladni. Subsequently other experimental researchers verified but did not explain this effect, notably Liskovius [20], Wertheim [21], Kundt [22, 23], Schneebeli [24], Dvorak [25] and others (see, e.g., Wood [26], pp. 249-257, 267-269). Lambosy [16] observed that 19th Century studies fall into two groups, those of physicists interested in acoustic phenomena and those of physiologists interested primarily in the pulsatile flow of blood, but “Helmholtz approached the study of acoustics ... largely from the physiological aspect” ([14], p. 83). It was Helmholtz (see, e.g., [14], pp. 43, 82-85; [26], p. 267; [16]; [27]) who not only brought to a peak the parallel strand of hydrodynamic theory of axial sonic waves in tubes developed by Poisson, Hopkins, Duhamel, Quet, et al, but also seems to have been the first to suggest in 1848 that the observed reductions in the speed of sound in tubes were due to the influence of the elasticity of the pipe walls (Helmholtz [28], pp. 242-246). He did not give a theoretical equation for this phenomenon and it was to be thirty years before Korteweg finally did, though in 1858 Ménébréa [29] appears to have independently come to the same conclusion as Helmholtz, as well as close to a theoretical model [30], as does Airy in the first (1868) edition of his book on sound [31]. Amongst physiologists, in 1850 Ernst-Heinrich Weber [32] stated that his brother Wilhelm had prepared a theory for the wave celerity which they found to be the same as the then forgotten result of Thomas Young [11, 12], though the theory itself did not appear until Wilhelm’s paper of 1866 [33] (see [16]). Weber [33] developed two separate equations for the elasticity of the pipe walls and the acceleration of the fluid column, and, combining these, he obtained a wave equation and consequently the expression for the wave celerity (for his modulus of elasticity $k = dR/dp$ where p is pressure and $R = D/2$ is pipe radius):

$$c_{\text{Weber}}^2 = D/(4k\rho_f) .$$

In modern notation with Young’s modulus E , pipe wall thickness e and diameter D (see [27], [34]) this gives the familiar expression for the wave celerity of incompressible fluid in an elastic pipe:

$$k = dR/dp = D^2/(4Ee) \quad \Rightarrow \quad c_1^2 = Ee/(\rho_f D) .$$

Subsequently there were a number of studies in this field, e.g. Donders, Landois, et al (see [16]). Marey (see [16] and [34]) conducted a series of excellent experiments on pulsatile flow in rubber tubes as a development of the Webers’ work and was able to accurately describe several features of the phenomenon, e.g. Marey [35]. However, he lacked the necessary mathematics to develop a theory so this was done for him by Résal, editor of the *Journal de Mathématiques Pures et Appliquées*. Résal [1, 2], apparently independently of any knowledge of the previous work of Young and Wilhelm Weber, rederived the wave equation and the associated expression for wave celerity in its modern form. It is against this historical background that the works of Moens and Korteweg can be assessed.

SUMMARY OF MOENS' DOCTORAL DISSERTATION

Moens worked from 1874 to 1877 on his doctoral dissertation under the supervision of Adrian Heynsius (see the Appendix for more biographical details) and successfully defended his achievements on 22 September 1877. An extended version of his thesis appeared one year later in the German language: "Die Pulscurve" (1878) [4]. This has been briefly, but adequately, dealt with by Dow [8]. All sections of his thesis (1877) [3] are reviewed below.

Introduction

The subject of Moens' doctoral dissertation is the study of the motion of a liquid in tubes with resilient walls. Despite the numerous previous investigations, while the laws of *flow* motion (velocity, internal pressure, etc) had, in the main, been discovered, this was not at all the case for the *wave* motion which can be generated in such tubes. Not only the period of oscillation and the speed of these waves but also the shape of them had not been satisfactorily explained.

Moens divided his experimental work in three parts: 1) Tap opening with trapped air (used in combination with a cardiograph to record pressure variations); 2) Tap closing with trapped air; 3) Wave propagation speed. Only Part 3 is reported in his thesis; Parts 1 and 2 were reported elsewhere (Chapters 2 and 3 in "Die Pulscurve", 1878, [4]). To simplify things in the Investigations 1 and 2, metal pipes were used with lumped elasticity represented by one or more air columns. Air, instead of rubber, was used because of its well-known elastic properties. Investigations 1 and 2 led to simple mathematical formulations describing the oscillatory phenomena. Rubber hoses were employed in Investigation 3.

Chapter 1 Historic review

Moens gives a structured review of previous work on the wave propagation speed in rubber and intestine tubes. This includes the well-known paper by Weber [32] and investigations by the physiologists Volkmann [36, 37], Donders [38], Ludwig [39, 40], Rive [41], Valentin [42], Landois [43] and Marey [44]. He neglects the contributions of Young (1808) ([11], [12]), possibly because of Young's "dunkele Darstellungsweise" ("obscure description") ([42], § 201). From this literature Moens distinguished seven factors that may influence the wavespeed, namely:

1) *Influence of static pressure:*

Donders, Rive and E-H Weber carried out experiments in rubber tubes where the wavespeed is of the order of 1 m/s and therefore relatively easy to measure. They observed lower wavespeeds and larger tube diameters (with hindsight, this causes lower wavespeed) at larger static pressures. Moens cites E-H Weber ([32], p. 182) who stated that according to his theory the wavespeed *increases* with diameter (with hindsight, maybe Weber misinterpreted his brother's formula [33]:

$$c_{\text{Weber}} = \sqrt{\frac{D}{4k\rho_f}}$$

by not realising that the elastic modulus, defined as $k = dR/dp$, in modern notation for circular pipes becomes $k = D^2/(4Ee)$, hence giving c_1). On the other hand, Marey and Weber (in intestine tubes) observed larger wavespeeds for larger pressures. There is some confusion here, because fluid pressure increases both tube diameter and axial (and hoop) tension in the tube wall, causing a membrane effect (mainly when using intestine tubes). In this respect, Moens correctly advises to keep both effects (tension in the tube wall and tube diameter) separated.

2) *Influence of tube diameter:*

Donders stated that there is no influence and Marey stated that there indeed is an influence, without telling whether an increase of D decreases or increases the wavespeed.

3) *Influence of Young's modulus:*

Donders, Valentin and Marey agreed that the wavespeed increases with E , but there was no quantitative result. Valentin stated that the wavespeed is proportional to the square root of E , by falling back on Newton's $\sqrt{(E/\rho_t)}$ for disturbances travelling in a medium of density ρ_t .

4) *Influence of wall thickness:*

Marey observed that the wavespeed increased with wall thickness.

5) *Influence of liquid mass density:*

Marey experimented with water and mercury and found that the wavespeed in mercury was 3.5 times lower than in water, which agrees surprisingly well with the square root of the ratio of mercury and water densities: $\sqrt{13.5} = 3.7$.

6) *Influence of excitation:*

Weber's and Donders' experiences in rubber hoses indicated that there is no influence of wave intensity.

7) *Influence of tube mass density:*

Valentin thought the wavespeed to be proportional to $\sqrt{(E/\rho_t)}$. With hindsight one might arrive at:

$$c_1 = \sqrt{\frac{E}{\rho_f} \frac{e}{D}} = \sqrt{\frac{E}{\rho_t}} \sqrt{\frac{e}{D}} \sqrt{\frac{\rho_t}{\rho_f}} = c_t \sqrt{\frac{e}{D}} \sqrt{\frac{\rho_t}{\rho_f}},$$

showing no influence of ρ_t . Korteweg [5, 6] dealt with this subject.

Moens' conclusion is that all the different experiments have shed no light on the matter, because so many factors have an influence and the difficulty of changing one without changing the other. For example, in rubber hoses, it is impossible to vary elasticity and diameter independently. Moens' idea was to separate elasticity from diameter by using metal (not rubber or intestine) pipes (for inertia) with air vessels (for elasticity). Progress is then made by linking theory and experiment.

Chapter 2 Derivation of c_{Moens}

§1 Preliminary considerations

Moens gives a detailed description and analysis of his experiments. He starts with a reservoir-valve-pipe system (Fig. 1) containing a metal pipe (not a rubber hose, like previous researchers). In steady state, the flow is turbulent, because of his assumption $v \sim \sqrt{p}$ (where v is fluid velocity and p is pressure), uniform and with a constant pressure gradient. In unsteady state he considers a rigid column with "living force" mv^2 (reflecting the 19th "vis viva" confusion between momentum and energy). Sudden valve closure leads to a shock which is more violent when the metal pipe is longer (larger value of $2L/c$, i.e. effectively more "rapid" closure), wider (contradictory if smaller v but discharge may have increased?) and the pressure gradient is larger (larger v). Vibrations of the liquid and the tube are felt (i.e. waterhammer). Valve closure gives a pressure drop behind the valve and a vacuum forms for large flow velocities (i.e. column separation). In the metal pipe c is hundreds of metres per second. Moens' invention is to add a short section of rubber tube as elastic factor between valve and pipe (Section MM_1 in Fig. 1). He gives a full and correct (waterhammer like) and very clear explanation of what is going on. Alternating suction and squeeze by the rubber (collapse and expansion) results in a rigid-column oscillation of the water in the metal pipe. The rubber "breathing" can be recorded by a cardiograph (connected to rubber section, not applicable to metal pipe). He makes three observations from the measured oscillations: 1) the amplitude decreases in time, 2) the amplitude depends on initial pressure-gradient, and 3) the period is independent of amplitude. Then follows a description of what is going to happen in an entirely rubber tube. In explaining (2D) wave propagation he follows the brothers Weber (1825) [45].

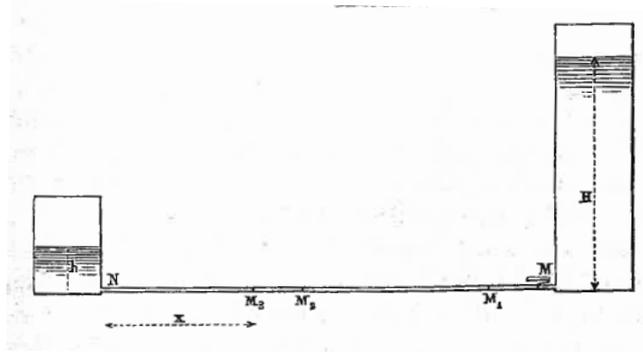


Figure 1. Moens' experiment.

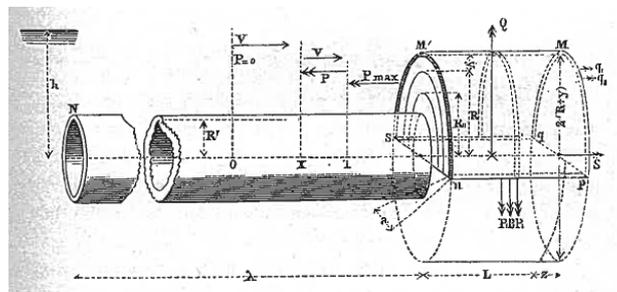


Figure 2. Moens' rigid metal pipe and elastic rubber section.

§2 Theoretical determination of period of oscillation T

Moens theoretical interpretation of his experiments proceeds through three stages:

- The system consisting of rigid column plus local elasticity (rubber section or equivalent air chamber) is depicted in Fig. 2. With hindsight, three critical remarks can be made with respect to Moens' analysis: there is 1) no Poisson contraction of the rubber section; 2) no longitudinal constraint, at least no influence of it (p. 22); and 3) no force on left, outer annular ring (not important if $R = R'$ as assumed later on). Basic principles of mechanics are applied, here in terms of kinetic energy of the liquid mass and work done by pressure in the rubber tube. The period of oscillation is calculated as:

$$T_{mr} = 2\pi \sqrt{\frac{\rho_f L_m L_r D_r}{E_r e_r}},$$

where the subscript r is for the rubber section and the subscript m for the metal pipe. The factor π comes from the integral of $1/\sqrt{1-x^2}$ and not from circle area or angular velocity. The mass of the liquid in the metal pipe is combined with the elasticity and storage of the rubber section. The above formula is valid if there is no longitudinal extension of the rubber section (which should be the case when it is rigidly

connected to the pressure vessel) in line with the footnote by Moens (p. 30). This formula is valid for a system where mass (inertia) and spring (elasticity) are separated. Now it is Moens' task to show that it is valid for a distributed system (rubber tube).

- b) Moens continues with inserting the elastic section at an arbitrary location along the metal pipe, as illustrated in Fig. 1, where the rubber section is between M_2 and M_3 in the metal pipe. The incompressible liquid in the rigid pipe between M_1 and M_2 stands still after valve closure, so that the formula for T_{mr} applies with the shorter length $x = M_3N$ and proportionally lower mass. The mass oscillation is at a higher frequency now and Moens nicely shows that this is equivalent to having an end rubber section of reduced length xL_r/L_m . He mentions a full experimental confirmation of his analysis (footnote on p. 31).
- c) Moens mimics an entirely rubber tube by connecting at regular intervals air columns to a metal pipe (like rigid-column with discrete gas cavities, DGCM) and in fact "discretises" the tube (like discrete FEM). First he inserts two elastic sections in the metal pipe (at MM_1 and M_2M_3 in Fig. 1), thus obtaining a two mass-spring system consisting of two rigid water columns and two elastic air pockets. Moens was not able to find a mathematical solution for it. However, experimentally he found the same period of oscillation (after a transient period) independent of the position of the two elastic sections. In addition he proved (theoretically and experimentally) that one end section can replace two arbitrary positioned sections by summing the two reduced lengths. As explained in his introduction, the experiments were with air columns instead of rubber sections. Additional advantages are the large amplitudes of oscillation and the large air volumes that are allowed, whereas only short rubber sections could be used to ensure uniform (lumped) behaviour. In the experiments the air volumes are precisely known and connected to a cardiograph for recording the oscillation. The theoretical period of oscillation is:

$$T_{ma} = 2\pi \sqrt{\frac{\rho_f L_m \varphi_a}{A_m p_a}},$$

where A is cross-sectional area, φ is air volume, p is air elasticity, and subscript a stands for air. Moens demonstrates the mathematical equivalence of air column (T_{ma}) and rubber section (T_{mr}). He carried out many experiments with one and two air columns connected to metal pipes of $L_m = 2.4$ m, $D_m = 19$ mm, and found convincing results. Experiments with m air columns (number not specified, but elastic tube is considered to be a metal pipe equipped with many (m) air columns) are possible and in this way can wave propagation be visualised. Actually, one can *see* wave propagation in a rubber hose. Here the rubber hose is 2.5 m long and equipped with one air column (number $m+1$) at 1.0 m from the outflow. The same result is obtained for a rubber hose with a smaller (factor 1.0/2.5) air volume φ_a placed at the valve.

More discrete-model experiments follow with the rubber tube, now with two air columns ($m+1$ and $m+2$) placed at the valve and at 1.0 m from the outflow, showing that Moens' reduced length theory also holds for rubber tubes. In summary, Moens uses a rigid pipe equipped with many air columns of the same volume and short distances apart to mimic an elastic tube. Each interior air column can then be replaced by one air column of reduced volume (or rubber section of equivalent reduced length) at the end of the tube. Thus taking infinitely many air columns and summing infinitely many reduced lengths, Moens finds that for an entirely elastic tube $L_r = L_m/2$ is the reduced length of the rubber end section replacing all the air columns. Substituting this and $L_m = L$ in T_{mr} gives the formula:

$$T_r = \pi L \sqrt{\frac{2\rho_f D_r}{E_r e_r}},$$

which is the mathematically derived period of oscillation of an entirely rubber tube. Moens showed theoretically and experimentally that his approach is right.

§3 Experimental determination of wavelength λ

The idea of a "breathing" rubber tube or membrane shown in Fig. 3 (left) is physically implemented in the laboratory system shown in Fig. 3 (right). It is a 2 m long wooden channel covered with a membrane and fully filled with water. The purpose is the visualisation of a $4L/c$ system. The system is excited by shortly lifting the membrane at the closed end M and measuring the natural oscillation by means of two cardiographs (sphygmographs) at the positions C and D. A tuning fork is used for time setting. Thus it is experimentally found that the wavelength $\lambda = 4L$. The same result is found for rubber hoses with $D = 16$ mm and $e = 1.7$ mm. In tubes with thicker walls, e.g. $e = 2.5$ mm, small deviations were found and Moens attributed these to the non-uniform stress distribution in the tube wall.

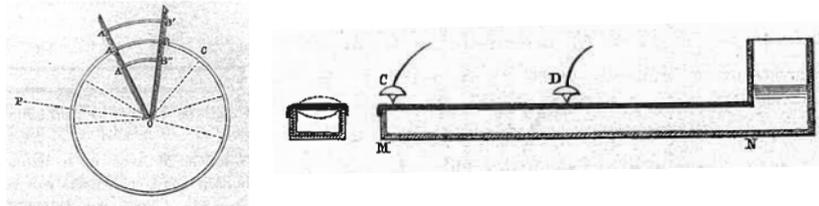


Figure 3. Moens' circular (left) and rectangular (right) conduits.

§4 Determination of wavespeed c

The wavespeed is now obtained by substituting $T = T_r$ in $c = \lambda/T = 4L/T$ (and omitting the subscript r) as

$$c_{\text{Moens}} = \alpha \sqrt{\frac{E e}{\rho_f D}},$$

with $\alpha = \frac{2\sqrt{2}}{\pi} = 0.9$. For pressure-dependent E , Moens prefers to use the formula in terms of strain,

$$c_{\text{Moens}}(h) = \alpha \sqrt{\frac{g \Delta h}{2 \Delta \varepsilon}}$$

by replacing E with the hoop stress-strain ratio $\frac{\Delta \sigma}{\Delta \varepsilon} = \frac{D \Delta p}{2e \Delta \varepsilon} = \frac{D \rho_f g \Delta h}{2e \Delta \varepsilon}$, where the

change in pressure head, Δh , causes a change in hoop strain, $\Delta \varepsilon$ (radial wall displacement divided by tube radius) (see Fig. 4). This is important for explaining the experiments on intestine tubes and on the aorta in Chapter 4. Compared to c_{1W} , it is seen that Moens' coefficient α is somewhat too small. It is not entirely clear to the authors where exactly the deviation comes from.

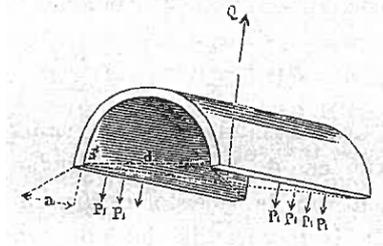


Figure 4. Moens' derivation of relation between fluid pressure and hoop stress.

Chapter 3 Discussion of c_{Moens}

The key result of the dissertation is

$$c_{\text{Moens}} = \alpha \sqrt{\frac{E e}{\rho_f D}},$$

which is valid for wave propagation in rubber and intestine tubes. The main problem is that D , e and E depend on internal pressure. Moens verified the four dependencies which in general explain previous measurements: 1) inversely proportional to $\sqrt{\rho_f}$ is entirely in agreement with Marey's 3.5 : 1 ratio mentioned before; 2) proportional to \sqrt{e} , though in rubber tubes under the same static pressure it is difficult to change e without changing D and E too; 3) inversely proportional to \sqrt{D} is consistent with E-H Weber's experimental results assuming that the product $D.e$ is constant when rubber tubes (with $\nu = 0.5$) expand under internal pressure; 4) proportional to \sqrt{E} with the difficulties arising from non-constant E explaining Weber's experimental results on intestine tubes. Viscoelastic effects and amplitude dispersion play a role.

Chapter 4 Measurement of c_{Moens}

In the validation of his formula for wavespeed, Moens uses the expression $c_{\text{Moens}}(h)$, because of the pressure dependence of D , e and E in intestine and (to a lesser extent) in rubber tubes and also because h and ε (dimensionless wall displacement) can be relatively easy measured in rubber hoses. His experimental parameters are determined as follows: ε from perimeter, D calculated from measured tube volume, e measured with micrometer, and E and ε directly measured from static tests.

§1 Constant E

The experiments concern a red vulcanised rubber tube with $D = 15.7$ mm and $e = 2.5$ mm. The mass density ρ_f for water is 1 gram per cubic centimetre. Dead-weight measurements on an 0.8 m long tube clearly showed a viscoelastic effect (instantaneous and retarded response). The estimated $D(h)$ and $e(h)$ are given, where h is load in terms of internal pressure head. The measured values of E and $\Delta\varepsilon$ were nearly constant. The predicted c_{Moens} for decreasing internal pressure is from 13.9 to 14.3 m/s, where the measured value in his own experiments was 14.3 m/s. Then he checks against Weber's experiments. Moens remarks that Weber ignored the viscoelastic effect (Kries remarked this too, see [46-48]). The too large $\Delta\varepsilon$ value of Weber was used in Moens' formula giving 8.5 m/s (experiment) versus 11.4 m/s (measured). Tests by Moens in a white

vulcanised rubber tube with $D = 13.1$ mm and $e = 2.5$ mm gave 14.2 m/s (theory) versus 16 m/s (experiment). In tests with very thin-walled tubes of 60 mm circumference, the tubes collapsed at zero pressure leaving behind a flat tire of 30 mm width.

§2 Variable E

Moens shows that in intestine (of a pig) tubes E increases with pressure and he measured the wavespeed. With $E(h)$ taken into account through $c_{\text{Moens}}(h)$ there was such a good agreement between theory and experiment (of course within specified measurement uncertainties) that Moens had no doubts about the correctness of his formula.

§3 In arteries

This is a description of the non-uniform properties and typical dimensions of arteries, things difficult to measure in those days. Moens presents his own dead-weight measurements of D , e and E of the aorta hoops of two humans and one pig. Young's modulus E and diameter D increase with pressure load, whereas thickness e decreases. The final conclusion is that E increases significantly with head h and so does c . This is opposite of what was found for rubber hoses. The speed of the pulse c is estimated for different types of arteries and found consistent with measurements by Weber.

§4 At low blood pressures

Tests on humans were carried out to show that the wavespeed decreases with blood pressure. The pulse was measured by two cardiographs at estimated distances connected to arteries and a tuning fork vibrating at 20 Hz for time ticking. During the measurement the test person was asked to keep his breath and to press, so to reduce the blood pressure. This indeed gave a lower wavespeed (about 10%). A second series of tests was on an "open" goat, which had the advantage of post-mortem measurements of artery lengths. Low blood pressure was obtained by inducing cardiac arrest and again a significantly lower (now more than 50%) wavespeed was observed. Thus it seemed possible to estimate blood pressure from wavespeed. The newly derived formula for c made it also possible to detect anomalies, although their dependence on three unknown factors (D , e and E) formed an intrinsic difficulty.

SUMMARY OF KORTEWEGS' DOCTORAL DISSERTATION

Korteweg successfully (cum laude) defended his thesis on 12 July 1878 and became the first doctor of the (newly) named University of Amsterdam (and not Leiden where his thesis was printed and published). He had worked on it for *one* year under the supervision of later Nobel Prize winner Van der Waals. During that time he was employed as a teacher at the Secondary Modern School of Breda (see the Appendix for more biographical details). Korteweg's thesis has no figures, but this is not amiss because everything is explained so clearly in words. He appears aware of the then available knowledge on (sound) waves (Airy, Helmholtz, Kirchhoff, Rayleigh, Weber) and elasticity (Lamé) and shows a clear interest in the many appearances of waves: shallow water waves, groups of waves, solitary waves, wave dispersion and attenuation, theory and experiment. The summary that follows is as literal as possible, but with changed notation (conventional modern waterhammer symbols).

Introduction

Korteweg gives a long introduction, which is a nice summary of the entire thesis, leaving all the mathematics to later chapters. The motive for his investigation is Moens' dissertation and a remark by Airy. It appeared to Korteweg that the formula for the speed

of propagation of the pulse (c_{Moens}) derived by Moens through combination of experiment and theoretical consideration, could also be derived along a purely mathematical path, but then freed of the empirical coefficient α . After indeed succeeding in that exercise, Korteweg read in Airy's book [31] on page 152: "Wertheim [21] has attempted an experimental determination of the velocity of sound (c_0) in (unconfined) water by immersing an organ-pipe in water; and forcing the water through it, a sort of musical tone was produced, sufficiently good to have its pitch recognised. The velocities found for water of the Seine varied from 1173 m/s to 1480 m/s; all much lower to those found by direct experiment. To reconcile them, Wertheim supposed that the velocities in a column of water and in unlimited space of water are not the same (as is the case in solids); an idea which we [i.e. Airy] do not accept. Among possible causes of the difference, we might suggest the yielding of the sides of the tube when pressed by the vibrations of a dense liquid [i.e. as Helmholtz suggested in 1848]". After this citation, Korteweg continues with explaining his idea of including both the elasticity of the tube and of the liquid in his mathematical analysis and how the limit cases c_0 (rigid pipe) and c_1 (rigid liquid) automatically follow from it. The mixed case where both tube and liquid elasticities are important should clarify the values of Wertheim's measured wavespeeds. Working out this idea, and reporting and validating it, is the aim of the dissertation. He considers all possible deformations of a tube and states, for example, that axi-symmetric bending stresses are only important at high frequencies. Korteweg introduces c_1 and names it *formula of Résal* (obtained in 1876 by pure mathematical derivation) and he lists five assumptions on which it is based. Résal's result [1, 2] was overlooked by the physiological community, including Moens, who derived the formula independently with semi-empirical factor α . Korteweg derived the formula as a special case of his more general formulas. Korteweg informed Moens about his own mathematical derivation. Later Korteweg found Résal's paper and directly forwarded it to Moens (see also the footnote by Moens in "Die Pulscurve" [4] on page 90). Much attention is paid to the experimental validation of the Résal-Moens-Korteweg formula. Previous work is critically reviewed including (long) citations of Savart, Weber, Helmholtz, Kundt and Moens. The focus (and a long discussion) is on Wertheim's physical experiments in view of Korteweg's theoretical results.

Chapter 1 Linear wave theory

This chapter is a textbook-style introduction to linear waves; complete, detailed and clearly written. Stating that $p/v = \text{constant}$ [i.e. ρc , but not stated as such by Korteweg] along a wave path, he defines the (so important) wave propagation speed as $\left(\frac{\partial x}{\partial t}\right)_{\left(\frac{v}{p}\right)}$,

where the subscript indicates the paths in the distance-time plane along which $v/p = \text{constant}$. Three representative examples of wave propagation are elaborated: 1D wave equation (no dispersion, no attenuation), Bernoulli-Euler beam (from Rayleigh [49]) (frequency dispersion) and 1D wave equation with damping term (attenuation).

Chapter 2 The pulse in thick-wall tubes

This chapter deals with incompressible liquid in an elastic tube and comprises a (short) mathematical derivation of the formula (c_1) of Résal (Korteweg names it after him):

$$c_1 = \sqrt{\frac{E' e}{\rho_f D}} = c_{\text{Resal}}.$$

Then follows a thorough analysis of what the value of E' should be (Korteweg's E and E' notation is swapped herein). Because the tube wall is not free but under internal

pressure, E' is not Young's modulus E per se, but it may include corrections due to Poisson contraction (and axial pipe restraints) and hoop and radial stresses that are not uniform in a thick wall. The analysis is based on Lamé's [50] pressurised hoop solutions that are in terms of the Lamé elasticity constants λ and μ . Korteweg neglects longitudinal tube inertia and longitudinal stresses (but later on finds that these are $\nu = 1/3$ of the hoop stresses). For a thick-walled tube he finds:

$$E' = E \left(1 - \psi \frac{e}{D} \right)$$

with $\psi = 3/2$ or $5/3$ according to either Poisson ($\lambda = \mu$) or Wertheim ($\lambda = 2\mu$). It is to be noted that around 1850 Poisson's ratio was thought of as general number instead of material property and its relation to the Lamé constants was not known, which makes translation to modern results difficult (but it is implicit in the results). Korteweg follows Wertheim ($\nu = 1/3$), so that:

$$E' = E \left(1 - 5\nu \frac{e}{D} \right).$$

If longitudinal wall motion is prohibited, then the (plane-strain hoop) correction should be applied:

$$E' = \frac{E}{1-\nu^2} \left(1 - \frac{2e}{D} \right).$$

The main conclusions are that thick-wall stresses decrease c_1 and full longitudinal restraint plane strain increases c_1 . Finally he shows that the energy distribution in the pulse wave is $1/2$ kinetic and $1/2$ potential (as it is in standard 1D wave propagation).

Chapter 3 Waterhammer with tube (hoop) vibration

This chapter introduces fluid elasticity in addition to tube wall (hoop) elasticity. The main result is a formula for the classical waterhammer wavespeed (later on used by Joukowski [51] in his famous investigation of waterhammer in the Moscow water distribution system):

$$\frac{1}{c_f^2} = \frac{1}{c_0^2} + \frac{1}{c_1^2} \quad \text{or} \quad c_f = \frac{c_0 c_1}{\sqrt{c_0^2 + c_1^2}} \quad \text{or} \quad c_f = \frac{c_0}{\sqrt{1 + \frac{e K}{D E}}} = c_{\text{Korteweg}}.$$

Korteweg continues with the incorporation of radial tube inertia – in (probably) the first investigation of FSI – thereby introducing the solid bar wavespeed:

$$c_t = \sqrt{\frac{E'}{\rho_t}}.$$

He arrives at the frequency-dependent phase velocity:

$$c_f^* = c_f \left[1 + \frac{1}{2} \left(\frac{c_f}{c_t} \right)^2 \left(\left(\frac{c_f}{c_0} \right)^2 - 1 \right) \left(\frac{\pi D}{\lambda} \right)^2 \right].$$

This is an unimportant correction for waterhammer in slender water-filled steel pipes, because the wavelength λ is of the order of pipe length L . However, it will cause frequency dispersion of steep wave fronts that have widths of the order of D . Korteweg notes a significant dispersive effect on air waves in rubber tubes, where $c_f > c_t$, and where a more complicated formula has to be used. As a special case, for incompressible fluid in an elastic tube, Korteweg finds:

$$c_1^* = \frac{c_1}{1 - \left(\frac{c_1}{c_f}\right)^2 \left(\frac{\pi D}{\lambda}\right)^2},$$

showing dispersion and an increase in phase velocity due to radial tube vibration (FSI).

Chapter 4 Added fluid mass

This chapter considers axial flow velocities that are not uniform in a cross section as a result of radial flow and radial pressure gradients. Radial-axial flow interaction takes place and there is a phase difference between velocities at the tube's central axis and wall (boundary layer effect). Added fluid mass influences the tube-wall's radial vibration. The inertia of the tube wall itself is neglected. The general formulas (48-50) show a solution without attenuation. For waterhammer the nonlinear dispersion-equation (53) needs to be solved numerically. For the pulse an analytical solution is found with an approximately linear radial-velocity distribution (v_r) and a parabolic axial-velocity profile (v_x) (due to radial wall motion, not skin friction) with amplitude ratio:

$$v_r'/v_x' = \pi D/(2\lambda).$$

For long wavelengths ($\lambda \gg D$) the wavespeed can be approximated by:

$$c_1^{**} = c_1 \left[1 - \left(\frac{\pi D}{4\lambda}\right)^2 \right],$$

from which it is seen that the phase velocity slows down because of radial fluid velocities (added mass). An interesting fact is that Korteweg finds in his harmonic analysis (on page 119) for the axial displacement amplitude d' of the fluid (κ is wave number and ω is angular frequency):

$$d' = \frac{\lambda}{2\pi c_f^2 \rho_f} p' = \frac{1}{\kappa c_f} \frac{1}{c_f \rho_f} p' = \frac{1}{\omega} \frac{1}{\rho_f c_f} p' \quad \text{or} \quad p' = \rho_f c_f \omega d' = \rho_f c_f v',$$

which is the Joukowsky formula in the frequency domain.

Chapter 5 Skin friction

This chapter gives 2D solutions for laminar flow and is largely based on a paper by Kirchhoff [52] on sound waves in air. Similar solutions form nowadays the basis for the modelling of unsteady friction. No-slip at the wall causes frequency-dependent boundary layers and the dimensionless number (on page 155):

$$\frac{D}{2} \sqrt{\frac{\omega}{\tilde{\nu}}}$$

is introduced, where $\tilde{\nu}$ is kinematic viscosity. This number carries many names, e.g. Stokes, oscillatory Reynolds, Womersley, Valensi, and more [53]. Governing 2D axisymmetric equations are derived from the full 3D Navier-Stokes equations and treated with harmonic analysis with complex numbers. The obtained dispersion relations show attenuation of propagating waves. The analytical solution is in terms of Bessel functions. Approximate solutions for long wavelengths are derived. Korteweg arrives at Kirchhoff's results (compressible flow without radial motion), that were not (yet?) validated by experiments. For the pulse (incompressible flow with radial motion) Korteweg finds the approximate solution for large Stokes number (on page 156):

$$c_1^{***} = c_1 \left[1 - \frac{1}{D} \sqrt{\frac{\tilde{\nu} \lambda}{\pi c_1}} \right] = c_1 \left[1 - \frac{1}{D} \sqrt{\frac{2\tilde{\nu}}{\omega}} \right].$$

The frequency-dependent attenuation k [in $\exp(-kx)$] is:

$$k = \frac{2}{D} \sqrt{\frac{\pi \tilde{\nu}}{\lambda c_1}} = \sqrt{\frac{2 \tilde{\nu} \omega}{D c_1}}.$$

Korteweg's approximate solution is valid for high frequencies (and thus not for the pulse in arteries) and he has some doubts himself (last sentence of thesis). This could be the reason that Chapter 5 does not appear in his German journal paper [6].

CONCLUSION – BEYOND KORTEWEG

The "Moens-Korteweg" formula has a long history with key milestones in the work of Young [11], Weber [33] and Résal [1, 2], as well as Moens [3, 4] and Korteweg [5, 6] themselves. While Young preceded anyone else (70 years ahead) [12], nevertheless it was Korteweg who mathematically derived the classical wavespeed that includes both liquid and pipe-wall elasticity, while Moens should be praised for his extensive, ingenious and accurate physical experiments in metal pipes equipped with air pockets which stimulated Korteweg's analysis. Korteweg's 1878 paper in German [6] is very well known and built upon by others. From the engineering waterhammer point of view, the key legacy of Korteweg was the contribution of Joukowsky [51] who referenced him (as well as Résal, Gromeka, Lamb and Ménabréa) in his seminal study, but this part of Joukowsky's work was not included in the English language account published by Simin (see [54]). Lamb [55] also went on to improve and extend Korteweg's theory from a hydrodynamic and acoustic point of view, a process that continued into the 20th Century through the work of Green [56], Gronwall [57], Biot [58], Lin and Morgan [59] and others. This particular line of development had little apparent influence on waterhammer analysis until Skalak (1956) [60, 61] which led to subsequent developments by, e.g., King and Frederick [62], Thorley [63] and beyond (see [64], [65]).

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Note

Gent (Dutch) = Ghent (English) = Gand (French)
 Leiden (Dutch) = Leyden (English) = Leyde (French)

APPENDIX Brief biographies of Isebree Moens and Korteweg

Adriaan **Isebree Moens** was born on 15 November 1846 in Zierikzee, a small town in Zeeland, the south-western part of The Netherlands. His father Jan Isebree Moens was mayor of nearby Bommenede en Bloois [66]. The common assumption [8] that Moens preferred to be known by his middle name – like E. Benjamin Wylie, C. Samuel Martin and M. Hanif Chaudhry – is not true. In 1824, his father Jan received – by Royal Decree – permission to bear the double surname Isebree Moens, where Isebree is the surname of his grandmother (Adriaan's great-grandmother). Adriaan attended Gymnasium (grammar school) in Zierikzee from 1860 to 1866 and he studied Engineering in Gent in nearby Belgium from 1866 to 1871. In 1871 he took up the study of Medicine in Leiden, where in 1874 he became Assistant in Pathology. After a third and life-threatening infection he switched in 1874 to Physiology. There he carried out his investigations on wave propagation in arteries culminating in his doctoral dissertation of 1877. After obtaining the degree of Doctor of Medicine, he decided to go back to Zeeland to become – like Thomas Young – a family doctor in Goes, where he was voted councillor in 1883 [67, 67a]. After the passing away of his former supervisor Adrian Heynsius, he was offered in 1885 the Chair in Physiology in Leiden [68] (taken by Einthoven in 1886), but he declined it to continue his work as general practitioner. After a long illness he died on 24 June 1891 in Goes, at the age of 44, leaving behind five daughters from two marriages. The main sources of Moens' biography are the obituary [69] written by Nobel Prize winner and inventor of the ECG, Willem Einthoven (1891), and the article by Yale Professor Philip Dow [8]. Genealogic data can be found on the internet. Other useful sources are the biographies of natives of Zeeland [70]; National Archive, The Hague; Royal Library, The Hague; Museum Boerhaave, Leyden.



Figure 5 Moens [71] and his doctoral certificate [72].

Diederik Johannes **Korteweg** was born on 31 March 1848 in 's Hertogenbosch, a town in the south of The Netherlands, close to Eindhoven. His father Adrianus Johannis Korteweg was a lawyer. After finishing primary and secondary education in his birth place, he started studying in 1865 at what now is Delft University of Technology. Lacking engineering skills and having more interest in mathematics, he ended up as teacher instead of engineer. From 1869 to 1881 he was teacher at secondary schools, first in Tilburg and later in Breda. During this period he was preparing for the entrance exam leading to university and obtained it in 1876. Then things went really fast: in 1877 he obtained his Bachelor degree (University of Utrecht) and in 1878 his Doctor degree (cum laude, University of Amsterdam). In 1881 he was appointed as professor in Mathematics, Mechanics and Astronomy in Amsterdam. In 1913 he stepped aside to make place for his pupil and Netherlands' most prominent mathematician ever, L.E.J. Brouwer. Probably his most famous scientific achievement is the discovery of a special type of stationary (solitary) wave travelling in a rectangular channel: the cnoidal wave. He solved the well-known problem of Huygens' "sympathie des horloges" (the tendency of two pendulum clocks to synchronize with opposite phases when suspended side by side). With Burgers and van der Pol he is the only Dutch scientist after whom a differential equation is named: the Korteweg-de Vries equation; these days of importance in plasma physics. He was for sixty years Member of the Royal Academy and for seventy years of the Dutch Mathematical Society. From 1911 to 1927 he edited the Oeuvres of Christiaan Huygens. He was an editor of the *Revue Semestrielle des Publications Mathématiques* (1892-1938) and of the *Nieuw Archief voor Wiskunde* (1897-1941). Korteweg died on 10 May 1941 (the same year that Allievi passed away) in Amsterdam at the age of 93. The main sources of this biography are the obituary written by Brouwer [73] and the publications [74-77a].



Figure 6 Three times Korteweg.