Travelling discontinuities in waterhammer theory – attenuation due to friction

D J LESLIE
Department of Civil Engineering, University of Dundee, UK
A S TIJSSELING
Department of Mathematics and Computing Science, Eindhoven University of Technology, The Netherlands

ABSTRACT

The classical waterhammer equations allow the propagation of jumps (discontinuities) in pressure and velocity. A newly derived (as far as the authors know) and exact formula describes the attenuation of these jumps as a result of quasi-steady Darcy-Weisbach friction. For large jumps in fluid velocity (compared to the steady-state velocity) the attenuation is a hyperbola in time, whereas for small jumps (disturbances) it is exponential in time. For jumps propagating into a region of steady flow the attenuation contains both hyperbolic and exponential components.

The newly derived formula has been successfully verified against numerical results from conventional MOC waterhammer programs.

The theoretical formula provides a criterion for assessing the importance of friction. It can be used for simple predictions of wave propagation in long transmission lines, in systems with high flow velocities and in highly viscous flows.

NOTATION

A, P	points in xt plane
C^+, C^-	characteristic lines
c	wave speed
$c(\omega)$	phase velocity
D	pipe diameter
f	Darcy-Weisbach friction factor
f^*	linear friction factor (Hz)
i	imaginary unit
K^*	waterhammer elasticity modulus
L	pipe length
P	pressure
q	attenuation factor
t	time
V	fluid velocity
x	axial direction
Δ	jump or discontinuity

ρ	mass density		Subscripts	
ν	kinematic viscosity		$+$	upper side of characteristic
ω	angular frequency		$-$	lower side of characteristic
MOC	method of characteristics		0	steady state or initial value
ODE	ordinary differential equation		eq	equivalent
			i	imaginary part
			r	real part

1 INTRODUCTION

The importance, the validity and the numerical approximation of the Darcy-Weisbach friction term in the classical waterhammer equations have been the subjects of investigation in numerous papers (e.g. [1–3]). Because the Darcy-Weisbach term is strongly non-linear in nature, most, if not all, of these papers concern *numerical* methods.

The present paper introduces an *analytical* approach based on the method of characteristics (MOC). A technique developed by Leonard and Budiansky [4] and Chou and Mortimer [5] for linear systems has been extended herein to account for the quadratic Darcy-Weisbach term.

The analytical method is introduced in Section 2 for linear systems. The quadratic friction term is treated in Section 3, where small and large waterhammer jumps are considered as special cases. Section 3.5 gives a simple formula for practical use. The importance and possible applications of the derived formulae are mentioned in Section 4.

The presented work is part of a research project on vibrational damping mechanisms in liquid-filled pipes [6].

2 WATERHAMMER WITH LINEAR FRICTION TERM

The waterhammer equations with a linear friction term are:

$$\begin{aligned} \frac{\partial V}{\partial t}+\frac{1}{\rho}\frac{\partial P}{\partial x}+f^*V=0 \\ \frac{\partial V}{\partial x}+\frac{1}{K^*}\frac{\partial P}{\partial t}=0 \end{aligned} \tag{1}$$

When the friction term is linear no assumptions and approximations are needed in what follows. Thus an elegant mathematical treatment is possible. The linear friction term appears in laminar flow and it also describes the damping of small disturbances on top of a steady turbulent flow. The values of f^* for these two cases are given by $f^* = 8\,\nu/R^2$ and $f^* = fV_0/D$, respectively, where ν is the kinematic viscosity and f is the (American) Darcy-Weisbach friction factor.

Using the MOC we can rewrite equations (1) in the form of ODEs which are valid along the characteristic lines:

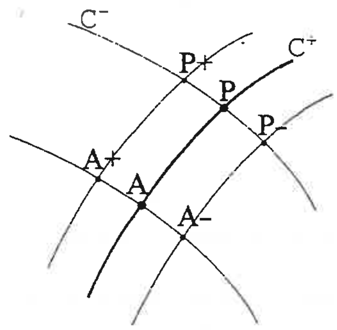

Figure 1: Notation on characteristic lines

$$\text{C}^{+}: \qquad \frac{dV}{dt} + \frac{1}{\rho c}\frac{dP}{dt} + f^{*}V = 0, \qquad \text{along } \frac{dx}{dt} = +c \tag{2}$$

and

$$\text{C}^{-}: \qquad \frac{dV}{dt} - \frac{1}{\rho c}\frac{dP}{dt} + f^{*}V = 0, \qquad \text{along } \frac{dx}{dt} = -c \tag{3}$$

where $c = \sqrt{(K^{*}/\rho)}$. We consider a discontinuity in V and P travelling along the characteristic line C^{+}. The pressure and velocity just ahead of the discontinuity (wave front) are denoted by V_{-} and P_{-} and those just behind the discontinuity by V_{+} and P_{+}. The discontinuity itself is $\Delta V = V_{+} - V_{-}$ for the velocity and $\Delta P = P_{+} - P_{-}$ for the pressure. The points A and P are on the characteristic line C^{+} carrying the discontinuity. Use A–, P– and A+, P+ for the associated points just ahead and just behind the discontinuity, respectively. This convention is clearly shown in Figure 1.

The relation between ΔV and ΔP is found by integrating equation (3) between the points P– and P+ on the C^{-} characteristic:

$$(V_{P+} - V_{P-}) - \frac{1}{\rho c}(P_{P+} - P_{P-}) + f^{*}\int_{P-}^{P+} V dt = 0 \tag{4}$$

and taking the limits: lim P+ → P, lim P– → P,

$$\Delta V - \frac{1}{\rho c}\Delta P = 0 \tag{5}$$

This is the well-known Joukowsky relationship, which is valid everywhere along the C^{+} characteristic.
Note that the friction term vanishes because the integration interval $\Delta t = t_{P+} - t_{P-}$ limits to zero. (In reality, wave fronts are not infinitely steep and the C^{-} integration interval Δt across the front will have a non-zero value. However, for steep wave fronts (jumps) the friction term in equation (4) is negligible because of the short lengths (rise times) involved. The friction term cannot be neglected in the remainder of this section, where jumps travelling long distances are considered.)

Returning to equation (2), describing wave propagation along the C$^+$ characteristics, we write integral forms for the upper and lower side of the jump-carrying characteristic:

$$(V_{P+} - V_{A+}) + \frac{1}{\rho c}(P_{P+} - P_{A+}) = -\int_{A+}^{P+} f * V dt \tag{6}$$

$$(V_{P-} - V_{A-}) + \frac{1}{\rho c}(P_{P-} - P_{A-}) = -\int_{A-}^{P-} f * V dt \tag{7}$$

and take the difference between these two equations:

$$(V_{P+} - V_{P-}) - (V_{A+} - V_{A-}) + \frac{1}{\rho c}(P_{P+} - P_{P-}) - \frac{1}{\rho c}(P_{A+} - P_{A-}) = -\int_{A+}^{P+} f * V dt + \int_{A-}^{P-} f * V dt \tag{8}$$

Now we take the limit as the upper and lower characteristic tend towards each other, i.e. $\lim \mathrm{P}+ \rightarrow \mathrm{P}$, $\lim \mathrm{P}- \rightarrow \mathrm{P}$, $\lim \mathrm{A}+ \rightarrow \mathrm{A}$ and $\lim \mathrm{A}- \rightarrow \mathrm{A}$. Thus (8) becomes:

$$(\Delta V)_P - (\Delta V)_A + \frac{1}{\rho c}(\Delta P)_P - \frac{1}{\rho c}(\Delta P)_A = -\int_{A=t_A}^{P=t_P} f * \Delta V dt \tag{9}$$

By taking the limit $\mathrm{A} \rightarrow \mathrm{P}$, after division by $\Delta t = t_P - t_A$, we obtain a differential equation for ΔV and ΔP:

$$\frac{d(\Delta V)}{dt} + \frac{1}{\rho c}\frac{d(\Delta P)}{dt} = -f * \Delta V \tag{10}$$

Equation (10) tells us that the compatibility equation (2) is valid not only for V and P, but also for ΔV and ΔP. Using the Joukowsky relationship (5), we may write (10) in terms of ΔP only, thus:

$$d(\Delta P) = -\frac{1}{2} f * \Delta P dt \tag{11}$$

which can be integrated to give:

$$\Delta P = (\Delta P)_0 e^{-\frac{1}{2} f * t} \tag{12}$$

where $(\Delta P)_0$ is the pressure jump at $t = 0$. Equation (12), which describes the attenuation of a pressure wave, is also found for jumps propagating along the C$^-$ characteristic.

2.1 Frequency-domain analysis

Frequency-domain analysis is possible for linear equations and in such an analysis exponentially decaying sinusoidal wave trains may be expressed as:

$$e^{i\omega(t - x/c(\omega))} = e^{-qx} e^{i\omega(t - x/c_{eq})} \tag{13}$$

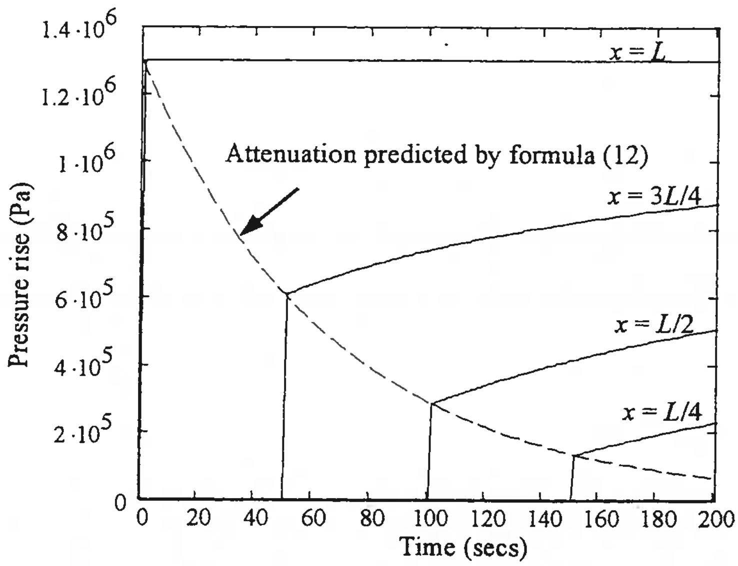

Figure 2: Solution of classical waterhammer in a single pipe (see Table 1) with linear friction. Initial steady-state values: $V_0 = 0$, $P_0 = 0$. Instantaneous velocity drop at downstream end: $(\Delta V)_0 = -1.3$ m/s. Solid lines: pressure histories at valve ($x = L$), three-quarter point ($x = 3L/4$), mid-point ($x = L/2$) and one-quarter point ($x = L/4$), calculated by MOC. Broken line: pressure rise at wave front as function of time, calculated from equation (12).

where the complex phase velocity $c(\omega) = c_r + \mathrm{i}\, c_i$ and ω is the angular frequency of oscillation. In equation (13) it is shown that the phase velocity may be written in the form of equivalent wave speed and attenuation factor as defined by, see [7, page 304]:

$$c_{eq} = \frac{c_r^{\,2} + c_i^{\,2}}{c_r} = \frac{|c(\omega)|^2}{c_r} \quad \text{and} \quad q = \frac{\omega c_i}{c_r^{\,2} + c_i^{\,2}} = \frac{\omega c_i}{|c(\omega)|^2} \tag{14}$$

For the waterhammer equations with a linear friction term, the phase velocities obtained from the dispersion equation are:

$$c(\omega) = c\left(1 + \frac{f^*}{\mathrm{i}\omega}\right)^{-½} \tag{15}$$

see [8, equation (17)]. From the equations (15) and (14) the attenuation factor in the high-frequency limit is determined as $q(\infty) = f^*/(2c)$, and also in this limit the equivalent wave speed $c_{eq}(\infty)$ equals the MOC wave speed c. Upon noting that $x = ct$ for the wave front, it is easily shown from (13) that in the high-frequency limit the attenuation is given by $\lim_{\omega \to \infty}[e^{-q(\omega)x}] = e^{-f^* t/2}$ agreeing with our previous result (12).

2.2 Waterhammer numerical simulation

Numerical solutions of the waterhammer equations with a linear friction term have been obtained with a Mathcad MOC program and the attenuation, as defined by the newly derived formula (12), was checked against the results. An example is given in Figure 2, clearly demonstrating that the formula does exactly predict the attenuation of the jump. Figure 2 illustrates the pressure history at intervals along the pipe so that at each location the arrival of the pressure jump is clearly visible. The dashed line superposed is the attenuation predicted by the formula (12).

The test problem is the single pipeline described by Kaplan *et al.* [9]. Table 1 gives its properties and the different test cases considered herein.

Table 1: Data used in examples

All examples:	$L = 201$ km	$D = 0.762$ m	$c = 1000$ m/s	$\rho = 1000$ kg/m^3
Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
$V_0 = 0$ m/s $(\Delta V)_0 = -1.3$ m/s $f^* = 0.03$ Hz	$V_0 = 1.3$ m/s $(\Delta V)_0 = \pm 1.3$ m/s $f = 0.018$	$V_0 = 1.3$ m/s $(\Delta V)_0 = -1.3$ m/s $f = 0.018$	$V_0 = 1.3$ m/s $(\Delta V)_0 = 1.3$ m/s $f = 0.018$	$V_0 = 1.3$ m/s $(\Delta V)_0 = -3$ m/s $f = 0.018$

3 WATERHAMMER WITH QUADRATIC FRICTION TERM

The classical waterhammer equations have a quadratic friction term:

$$\begin{gathered} \frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{f}{2D}|V|V = 0 \\ \frac{\partial V}{\partial x} + \frac{1}{K^*}\frac{\partial P}{\partial t} = 0 \end{gathered} \qquad (16)$$

To start off, it is assumed that V does not change sign so that V^2 is written in place of $|V|V$ and to account for this assumption we employ the convention that f is positive when $V_0 > 0$ and f is negative when $V_0 < 0$. However, the methodology presented can also be applied to situations in which the direction of flow changes during the passage of the jump, as will be shown in Section 3.4.

Using the MOC we can rewrite equations (16) in the form of ODEs which are valid along the characteristic lines:

$$C^+: \quad \frac{dV}{dt} + \frac{1}{\rho c}\frac{dP}{dt} + \frac{f}{2D}V^2 = 0, \quad \text{along} \quad \frac{dx}{dt} = +c \qquad (17)$$

and

$$C^-: \quad \frac{dV}{dt} - \frac{1}{\rho c}\frac{dP}{dt} + \frac{f}{2D}V^2 = 0, \quad \text{along} \quad \frac{dx}{dt} = -c \qquad (18)$$

These equations may be found in many texts, for example Wylie and Streeter [7, page 39]. The propagation of a discontinuity along the C^+ characteristic is again considered (see Figure 1) and the analysis proceeds parallel to that in Section 2.

It may be shown that the Joukowsky relationship (5) holds independently of the friction term employed and can be determined from (18). Thus as before we write integral forms of (17) for the upper and lower side of the C^+ characteristic and take the difference between the resulting equations. The upper and lower characteristic are allowed to tend towards each other and the $\lim A \to P$ is taken after division by Δt. In place of (10), the resulting equation is determined as:

$$\frac{d(\Delta V)}{dt}+\frac{1}{\rho c}\frac{d(\Delta P)}{dt}=-\frac{f}{2D}\Delta\left(V^2\right) \tag{19}$$

We may write $\Delta(V^2)$ in many different forms:

$$\begin{aligned}\Delta\left(V^2\right)&=V_+^{\,2}-V_-^{\,2}=\left(V_-+\Delta V\right)^2-V_-^{\,2}=V_+^{\,2}-\left(V_+-\Delta V\right)^2=(V_++V_-)(V_+-V_-)\\&=2V_-\Delta V+\left(\Delta V\right)^2=2V_+\Delta V-\left(\Delta V\right)^2=(V_++V_-)\Delta V\end{aligned} \tag{20}$$

from which we see that equation (19), using (5), can be integrated analytically when V_-, V_+ or $V_+ + V_-$ are constant. In general V_- and/or V_+ depend on t. Approximations which allow analytical integration can be made when the jump is either large or small, i.e. when $|V_-| << |\Delta V|$ (or $|V_+| << |\Delta V|$) and when $|V_-| >> |\Delta V|$ (or $|V_+| >> |\Delta V|$). The case when V_- is constant will be examined in detail and also cases for small and large jumps. Large jumps changing the sign of V are considered in Section 3.4.

Using the Joukowsky relationship (5) and one specific form of (20), we may write (19) in terms of ΔV only, thus

$$d\left(\Delta V\right)=-\frac{f}{2D}V_-\Delta V dt-\frac{f}{4D}\left(\Delta V\right)^2 dt \tag{21}$$

For large jumps, the second term in the right-hand side dominates, whereas for small jumps it is the first term in the right-hand side that is important.

3.1 Constant V_-

This case relates to the situation in which the jump is propagating into a region with a constant velocity, for example a region of steady flow. With V_- constant equation (21) can be integrated analytically, such that for a discontinuity propagation along the C^+ characteristic:

$$\frac{\Delta P}{(\Delta P)_0}=\frac{\Delta V}{(\Delta V)_0}=\frac{e^{-\frac{1}{2D}fV_-t}}{1+\frac{(\Delta V)_0}{2V_-}\left(1-e^{-\frac{1}{2D}fV_-t}\right)} \tag{22}$$

If the discontinuity was assumed to propagate along the C^- characteristic the analysis may be repeated giving an identical result to that above, noting that the Joukowsky formula as stated by (5) is replaced by a similar formula valid everywhere along the C^- characteristic:

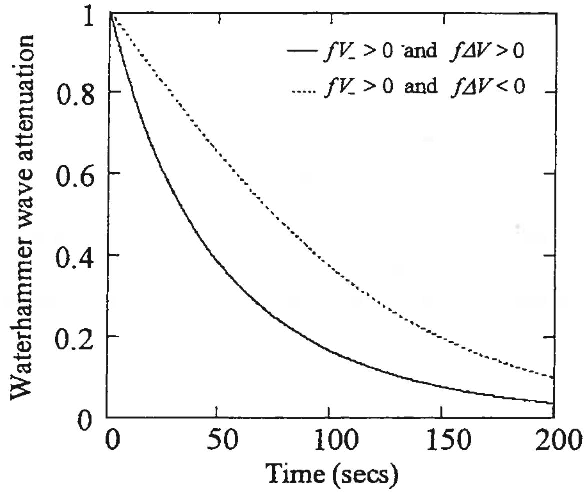

Figure 3: Comparison of attenuation factors given by the right-hand side of equation (22). Waves generated by a positive jump in flow velocity are damped more heavily than waves generated by a negative jump.

$$\Delta V + \frac{1}{\rho c}\Delta P = 0 \tag{23}$$

Taking into account possible cases that the formula (22) can be applied, there are two distinct attenuations. These are illustrated in Figure 3.

The initial propagation of a jump, for example following the sudden closure of a valve in a stationary reservoir-pipe-valve system, are examples in which V_- is constant and in particular for these cases $V_- = V_0$. Figure 4 illustrates the same example used in the paper by Kaplan *et al.* [9, Fig. 5; also in 7, Fig. 3-18]. Imperial units have been retained to allow direct comparison with the figure given in [9], but SI units are given in Table 1. The discontinuity propagates along the C^- characteristic. The attenuation is given by (22) noting that $\Delta V < 0$. Figure 5 gives an example with the propagation of the discontinuity along the C^+ characteristic, because the valve manipulation is now upstream and $\Delta V > 0$. The figure illustrates pressure histories at intervals along the pipe. Both figures confirm equation (22).

3.2 Small jumps

For small disturbances propagating in a steady flow, $V_+ \approx V_- \approx V_0$, we obtain from (19), (20) and (5):

$$\frac{d(\Delta V)}{\Delta V} = -\frac{f}{2D}V_0 dt \tag{24}$$

and integrate this linear equation:

$$\Delta V = (\Delta V)_0 e^{-\frac{1}{2D}fV_0 t} \tag{25}$$

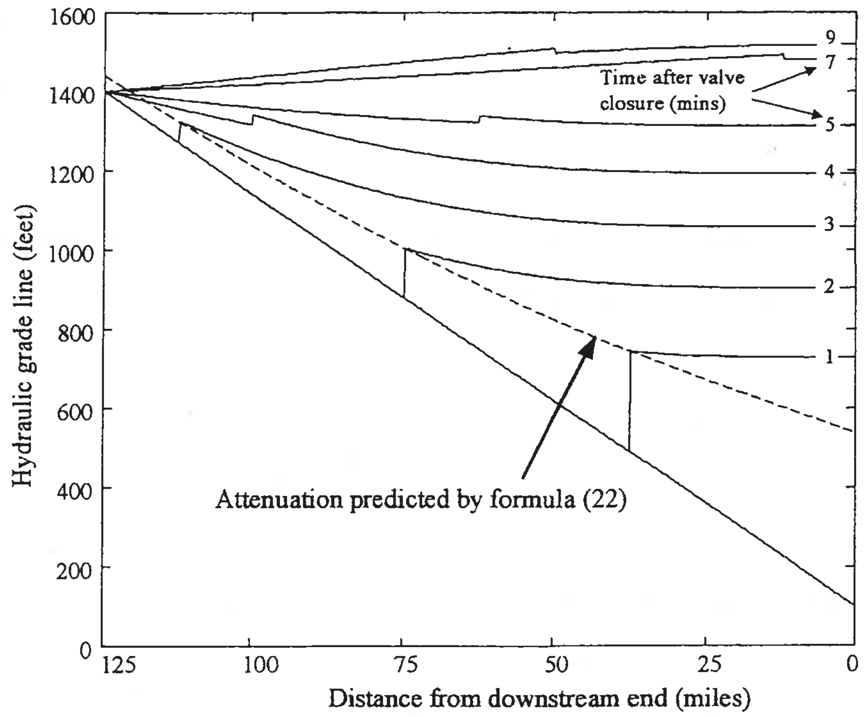

Figure 4: Solution of classical waterhammer in a single pipe with quadratic friction term. Valve closure downstream. Hydraulic grade line along the pipe is given at specified time intervals.

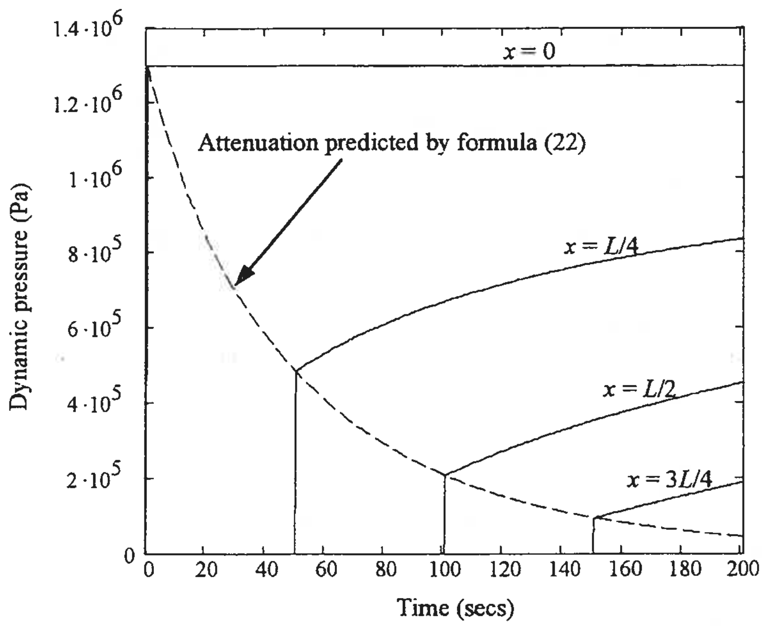

Figure 5: Solution of classical waterhammer in a pipe with quadratic friction term. Valve opening upstream giving instantaneous velocity increase at upstream end: $(\Delta V)_0 = 1.3$ m/s. Solid lines: Dynamic pressure histories at intervals along the pipe. Broken line: pressure rise at wave front as function of time, calculated from equation (22).

Because this is a linear friction case (Section 2), the decay in the jump is described by an exponential factor. Note that equation (25) directly follows from equation (22) by taking the limit $(\Delta V)_0 \to 0$. It also follows from (22) for large t, because eventually the jump will be small.

3.3 Large jumps

For large disturbances we obtain from (21) the approximation:

$$\frac{d(\Delta V)}{(\Delta V)^2} = -\frac{f}{4D}dt \tag{26}$$

Integrating this equation gives:

$$\Delta V = \frac{(\Delta V)_0}{1 + \frac{1}{4D} f (\Delta V)_0 t} \tag{27}$$

Note that equation (27) directly follows from equation (22) by taking the limit $V_- \to 0$. When $V_- = 0$, a jump is always large.

3.4 Jumps changing the flow direction

If a jump changes the direction of flow, the friction term in equation (16) changes sign and the right-hand side of equation (19) becomes $f/(2D)(V_+^2 + V_-^2)$ which equals $f/(2D)\{(\Delta V)^2 + 2V_-\Delta V + 2V_-^2\}$. Using equation (5) to eliminate ΔP the following ODE for ΔV is obtained:

$$\frac{d(\Delta V)}{dt} = \frac{f}{4D}\left\{(\Delta V)^2 + 2V_-\Delta V + 2V_-^2\right\} \tag{28}$$

which, if V_- is constant, can be integrated analytically:

$$\Delta V = -V_-\left[1 + \tan\left\{\arctan\left(-1 - \frac{(\Delta V)_0}{V_-}\right) - \frac{f}{4D}V_- t\right\}\right] \tag{29}$$

Equation (29) is valid as long as $t < t_c$ with t_c defined by:

$$t_c = \frac{4D}{fV_-}\arctan\left(-1 - \frac{(\Delta V)_0}{V_-}\right) \tag{30}$$

The wave front attenuates and at $t = t_c$ the jump has become so small that it does not change the flow direction anymore. For $t > t_c$ formula (22) is valid. Figure 6 illustrates an example where this is the case, i.e. the jump in the velocity is negative with a magnitude greater than the steady-state velocity. Attenuations given by (29), starting at $t = 0$, and (22), starting at $t = t_c$, are superposed on the numerical simulation.

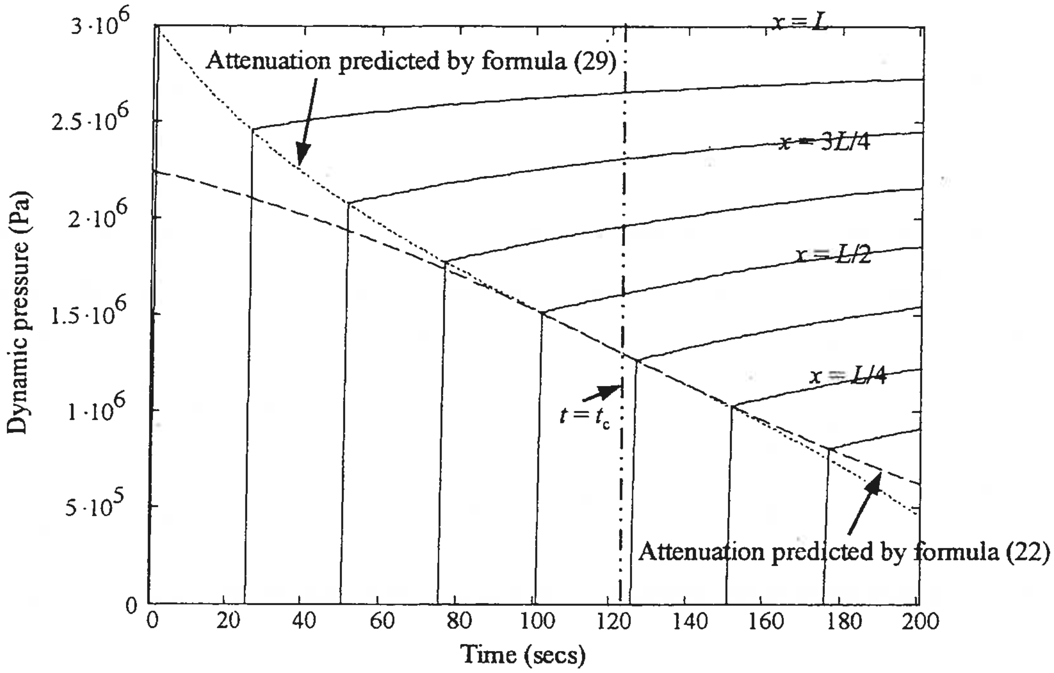

Figure 6: Solution of classical waterhammer in a single pipe with quadratic friction term. Special case in which a downstream jump changes the flow direction.

3.5 Extended Joukowsky equation

For those having done a steady-state analysis but not willing to do a full waterhammer analysis, we propose a useful extension to Joukowsky's equation, which takes into account the influence of quasi-steady Darcy-Weisbach friction. The Joukowsky formulae (5) or (23) are multiplied with the attenuation factor (22) so that:

$$\Delta P(x) = \pm \rho c\left(\Delta V\right)_0 \frac{e^{-\frac{f V_0 x}{2 c D}}}{1+\frac{(\Delta V)_0}{2V_0}\left\{1-e^{-\frac{f V_0 x}{2 c D}}\right\}} \tag{31}$$

where x is the distance travelled by the wave front and f, D and V_0 are additional parameters already known from the steady-state analysis.

4 CONCLUSIONS

The paper gives a mathematical derivation of formulae describing the attenuation of travelling waterhammer jumps as a result of quasi-steady Darcy-Weisbach friction. The formulae have been successfully checked against numerical results.

The derivation for linear friction requires no further assumptions, but for quadratic friction the derivation is valid only under certain conditions. Fortunately, these conditions are not severe. For example, the attenuation of a jump travelling into a region of steady flow can be calculated exactly.

The attenuation formulae may be used explicitly in numerical schemes to prescribe exact front propagation and so to increase computational accuracy and/or to reduce grid sizes.

The attenuation formulae may be used to assess the error caused by numerical integration of the friction term.

The attenuation formulae may be used to predict the importance of friction.

The extended Joukowsky equation (31) takes into account the influence of friction. It may be used in a typical reservoir-pipe-valve system to estimate the initial pressure rise not only at the closing valve, but also at any position along the pipeline. However, it does not include the effect of line packing.

It is noted that the magnitude and steepness of pressure jumps are important factors in fluid-structure interaction (FSI) analyses, because they determine the dynamic loads on the pipes.

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