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# FSI IN L-SHAPED AND T-SHAPED PIPE SYSTEMS

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## ABSTRACT

Vibrating elbows and tee-pieces are the primary sources of fluid-structure interaction (FSI) in liquid-filled pipe systems; they couple the dynamic behaviour of liquid and pipes. In particular, they alter the natural frequencies of the pipe system, which can be crucial for its resonance behaviour.

In this paper FSI is considered in the frequency domain. The theoretical development including governing equations, FSI sources at bends and branches, and solution method, has been dealt with in previous papers, but the important relations describing FSI at an elbow and at a tee-piece are given herein. New experimental results concern a freely suspended L-shaped pipe system. In contrast to experiments previously reported in literature, the system does not suffer from unknown support conditions. The experimental results for a T-shaped pipe system were taken from an earlier publication.

The natural frequencies observed in the L- and T-shaped systems are consistent with the theoretical predictions.

**Keywords:** FSI, structural vibration, waterhammer, experiment

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## 1. INTRODUCTION

The axial motion of pipe junctions like L-bows and T-pieces may cause significant fluid-structure interaction (FSI) in fast transient events such as waterhammer and structural impact. This phenomenon has been extensively studied in the time domain in previous work [Tijsseling *et al* 1996; Vardy *et al* 1996]. The present paper focuses on the role of FSI in the free and forced vibrations of liquid-filled pipe systems. The theoretical analysis is in the frequency domain and the experimental results concern the L- and T-shaped systems described in the two aforementioned references.

The Ph.D. theses of Lesmez [1989], Tentarelli [1990], Frikha [1992], de Jong [1994] and Svingen [1996] deal with the theoretical and experimental aspects of FSI in the frequency domain. The lists of references in these five theses cover most of the important literature on the subject. More general reviews are those by Wiggert [1996], Tijsseling [1996], Zhang *et al* [2000] and Wiggert & Tijsseling [2001].

The review papers show that many physical experiments have been performed in systems with elbows, ranging from Blade *et al* [1962] to Jiao *et al* [1999], and just a few in systems with branches. Nearly all of the experimental systems had at least one "fixed" end (support, anchor), for example the connection of the test pipe to a liquid supply (reservoir). "Fixed" stands for infinitely large impedance (zero mobility), something impossible in practice. Some researchers, like Davidson & Samsury [1972] and de Jong [1994], measured the mobility of the pipe supports in their test systems, but it is nevertheless common practice not to measure or estimate the mobility of supports, but to neglect it on the simplifying assumption that the support is (looks) rigid. Consequently, many investigators have overlooked support mobility. The well known experiments by Swaffield (196~~9~~8-1969) and Davidson & Smith (1969), the results of which have been used by many others, suffer from the ignored vibration of "fixed" points, as noted by Wilkinson [1980, p. 197] and Brown & Tentarelli [1988, p. 148], respectively. Svingen [1996, p. 76] reported "unintentional" valve motion. Care has to be taken in this respect, always. In contrast to the many other test rigs, the experimental apparatus employed herein has no "fixed" points at all, it is structurally "free".

The main contribution of the present paper is in the newly obtained experimental data for an L-shaped system without "fixed" points, that is: a single-elbow system without unknown support conditions. Frequency spectra have been deduced from impact tests. The measured natural frequencies are compared with theoretical predictions. Experimental results for a T-shaped system, taken from Vardy *et al* [1996], are also compared with theoretical predictions.

Moussou *et al* [2000] presented a very nice and detailed analysis of a two-elbow pipe system.

## 2. EXPERIMENTS

### L-shaped system

The L-shaped system consists of two steel pipes connected by a rigid 90 degrees elbow. The system is closed at its two ends and filled with water of at least 20-bar pressure. Three long thin steel wires carry the system and they practically allow free motion in a nearly horizontal plane. The system is excited by the axial impact of a long steel rod or by the lateral impact of a steel hammer. The dimensions and material properties of the pipes are given in Tables 1. Further details of the experiment can be found in [Tijsseling *et al* 1996].

Typical measured time histories, and frequency spectra derived from these, are shown in the Figures 1 and 2. The measuring time was 1.5 seconds during which, per gauge, 15000 pressures or axial strains were recorded.

### T-shaped system

The T-shaped system is symmetric and can be regarded as the L-shaped system, described above and in Tables 1, mirrored in the axis of the long pipe. A rigid tee-piece connects three steel pipes. The system is suspended in wires and excited by axial rod impact of the long leg. Further details of the experiment can be found in [Vardy *et al* 1996].

| Steel pipes | |
|---|---|
| pipe lengths | $L_1 = 4.51$ m<br>$L_2 = 1.34$ m<br>( $L_3 = 1.34$ m ) |
| inner radius | $R = 26.01$ mm |
| wall thickness | $e = 3.945$ mm |
| Young modulus | $E = 168$ GPa |
| shear modulus | $G = 65.1$ GPa |
| mass density | $\rho_t = 7985$ kg/m$^3$ |
| Poisson ratio | $\nu = 0.29$ |
| shear coefficient | $\kappa^2 = 0.53$ |
| end mass at $z$=0 | $m_0 = 1.312$ kg |
| end mass at $z$=$L_2$ | $m_L = 0.3258$ kg |
| area | $A_t = 694$ mm$^2$ |
| moment of area | $I_t = 272900$ mm$^4$ |

| Water | |
|---|---|
| bulk modulus | $K = 2.14$ GPa |
| mass density | $\rho_f = 999$ kg/m$^3$ |
| flow area | $A_f = 2125$ mm$^2$ |

TABLES 1. Geometrical and material properties of Dundee test pipes

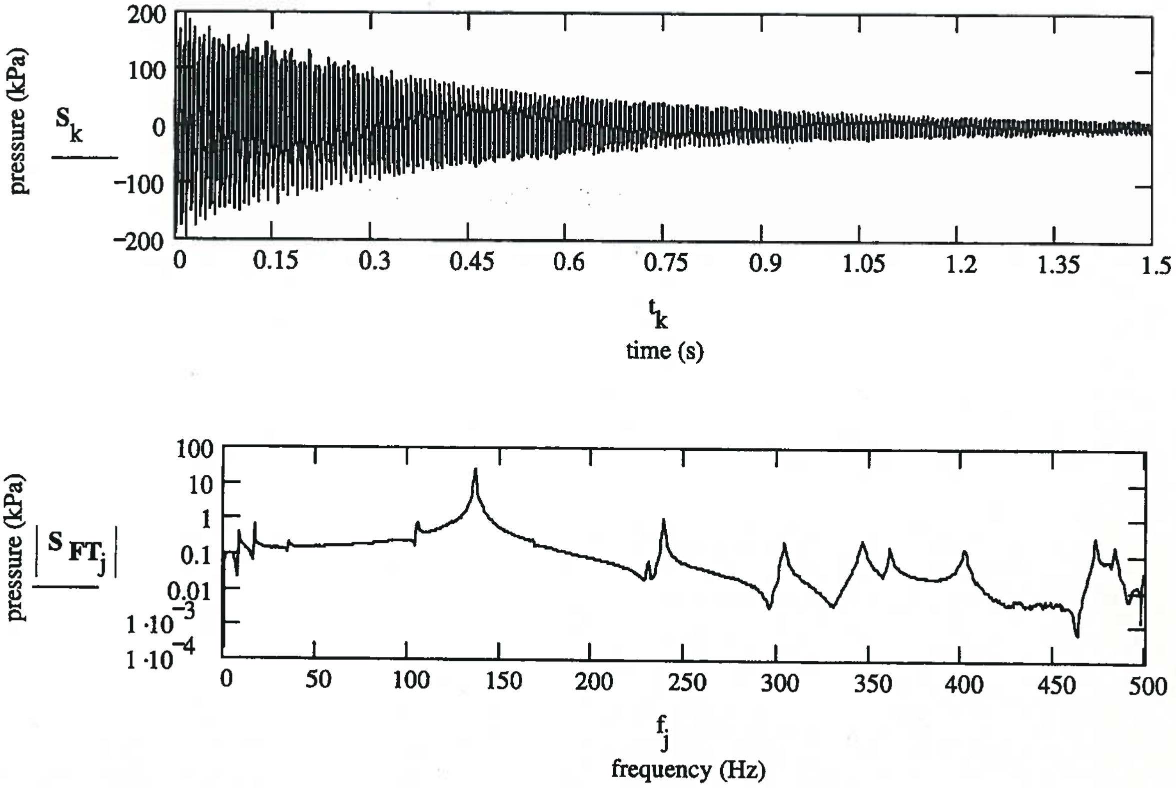

FIGURE 1. Experimental results: L-shaped system, axial rod impact, pressure near elbow.

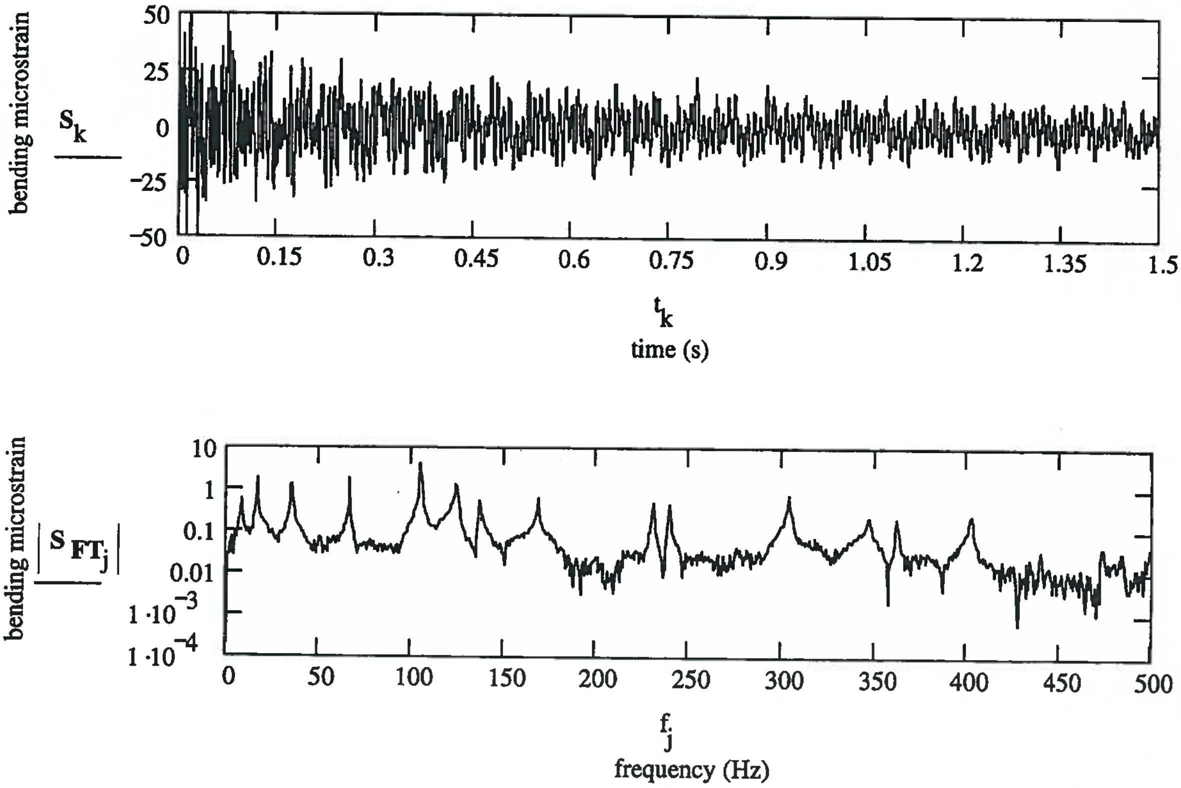

FIGURE 2. Experimental results: L-shaped system, lateral hammer impact, bending strain in short pipe.

## 3. THEORY

The theory underlying the predictions presented in Section 4 has extensively been dealt with in previous publications [Tijsseling *et al* 1997; Tijsseling & Vaugrante 1999; Zhang *et al* 1999] and is not repeated here. The Transfer Matrix Method has been applied to one-dimensional liquid-filled Timoshenko-type pipes. Acoustic transfer relations, junction coupling relations and boundary conditions, describing the entire pipe system and all FSI mechanisms, have been assembled in one global matrix, which multiplies the vector of the unknown dynamic variables such that the product equals the excitation vector. The FSI junction relations used for the elbow and the tee-piece are essential and therefore given in general form below.

### Elbow

The 14 junction relations for the three-dimensional vibration of an unrestrained elbow (mitre bend), including fluid-structure coupling, are:

(relative fluid velocity)

$$\{\ A_f\,(V-\dot{u}_z)\ \}_1=\{\ A_f\,(V-\dot{u}_z)\ \}_2 \tag{1a}$$

(pressure)

$$\{\ P\ \}_1=\{\ P\ \}_2 \tag{1b}$$

(axial velocity in pipe 1)

$$\{\ \dot{u}_z\ \}_1=\{\ \dot{u}_z\ \}_2\ \cos\alpha+\{\ \dot{u}_y\ \}_2\ \sin\alpha \tag{1c}$$

(axial force in pipe 1)

$$\{\ A_f\,P-A_t\,\sigma_z\ \}_1=\{\ A_f\,P-A_t\,\sigma_z\ \}_2\ \cos\alpha+\{\ Q_y\ \}_2\ \sin\alpha \tag{1d}$$

(in-plane lateral velocity in pipe 1)

$$\{\ \dot{u}_y\ \}_1=\{\ \dot{u}_y\ \}_2\ \cos\alpha-\{\ \dot{u}_z\ \}_2\ \sin\alpha \tag{1e}$$

(in-plane lateral force in pipe 1)

$$\{\ Q_y\ \}_1=\{\ Q_y\ \}_2\ \cos\alpha-\{\ A_f\,P-A_t\,\sigma_z\ \}_2\ \sin\alpha \tag{1f}$$

(in-plane angular velocity)

$$\{\ \dot{\theta}_x\ \}_1=\{\ \dot{\theta}_x\ \}_2 \tag{1g}$$

(in-plane bending moment)

$$\{\ M_x\ \}_1=\{\ M_x\ \}_2 \tag{1h}$$

(out-of-plane lateral velocity)

$$\{\ \dot{u}_x\ \}_1=\{\ \dot{u}_x\ \}_2 \tag{1i}$$

(out-of-plane lateral force)

$$\{\ Q_x\ \}_1=\{\ Q_x\ \}_2 \tag{1j}$$

(out-of-plane angular velocity in pipe 1)

$$\{\ \dot{\theta}_y\ \}_1=\{\ \dot{\theta}_y\ \}_2\ \cos\alpha+\{\ \dot{\theta}_z\ \}_2\ \sin\alpha \tag{1k}$$

(out-of-plane bending moment in pipe 1)

$$\{\ M_y\ \}_1=\{\ M_y\ \}_2\ \cos\alpha+\{\ M_z\ \}_2\ \sin\alpha \tag{1l}$$

(torsional angular velocity in pipe 1)

$$\{\ \dot{\theta}_z\ \}_1=\{\ \dot{\theta}_z\ \}_2\ \cos\alpha-\{\ \dot{\theta}_y\ \}_2\ \sin\alpha \tag{1m}$$

(torsional moment in pipe 1)

$$\{\ M_z\ \}_1=\{\ M_z\ \}_2\ \cos\alpha-\{\ M_y\ \}_2\ \sin\alpha \tag{1n}$$

where $z$ indicates the axial direction, $x$ and $y$ indicate the lateral directions, and $A_f$ and $A_t$ are the cross-sectional areas of flow and pipe (tube), respectively. The angle $\alpha$ is the change in flow direction and the indices 1 and 2 refer to either side of the elbow. The mass and dimensions of the elbow are neglected, just as the forces due to change in liquid momentum, which is consistent with the acoustic approximation. This simple model is valid if the length of the elbow is small compared to the lengths of the adjacent pipes. The angle $\pi-\alpha$ between the pipes remains constant; elbow ovalization and the associated flexibility increase and stress intensification are ignored. However, these matters can be accounted for by flexibility and stress-intensification factors.

The special case $\alpha = 0$ describes a diameter change (sudden pipe expansion / contraction).

It is noted that for the planar systems considered herein out-of-plane motion is disregarded.

### T-piece

The 21 junction relations for the three-dimensional vibration of an unrestrained T-piece, including fluid-structure coupling, are:

(relative fluid velocity)

$$\{\, A_f\,(V - \dot{u}_z)\,\}_1 = \{\, A_f\,(V - \dot{u}_z)\,\}_2 + \{\, A_f\,(V - \dot{u}_z)\,\}_3 \tag{2a}$$

(pressure)

$$\{\, P\,\}_1 = \{\, P\,\}_2 = \{\, P\,\}_3 \tag{2b}$$

(axial velocity in pipe 1)

$$\{\, \dot{u}_z\,\}_1 = \{\, \dot{u}_y\,\}_2 = \{\, -\dot{u}_y\,\}_3 \tag{2c}$$

(axial force in pipe 1)

$$\{\, A_f\,P - A_t\,\sigma_z\,\}_1 = \{\, Q_y\,\}_2 - \{\, Q_y\,\}_3 \tag{2d}$$

(in-plane lateral velocity in pipe 1)

$$\{\, \dot{u}_y\,\}_1 = \{\, -\dot{u}_z\,\}_2 = \{\, \dot{u}_z\,\}_3 \tag{2e}$$

(in-plane lateral force in pipe 1)

$$\{\, -Q_y\,\}_1 = \{\, A_f\,P - A_t\,\sigma_z\,\}_2 - \{\, A_f\,P - A_t\,\sigma_z\,\}_3 \tag{2f}$$

(in-plane angular velocity)

$$\{\, \dot{\theta}_x\,\}_1 = \{\, \dot{\theta}_x\,\}_2 = \{\, \dot{\theta}_x\,\}_3 \tag{2g}$$

(in-plane bending moment)

$$\{\, M_x\,\}_1 = \{\, M_x\,\}_2 + \{\, M_x\,\}_3 \tag{2h}$$

(out-of-plane lateral velocity)

$$\{\, \dot{u}_x\,\}_1 = \{\, \dot{u}_x\,\}_2 = \{\, \dot{u}_x\,\}_3 \tag{2i}$$

(out-of-plane lateral force)

$$\{\, Q_x\,\}_1 = \{\, Q_x\,\}_2 + \{\, Q_x\,\}_3 \tag{2j}$$

(out-of-plane angular velocity in pipe 1)

$$\{\, \dot{\theta}_y\,\}_1 = \{\, \dot{\theta}_z\,\}_2 = \{\, -\dot{\theta}_z\,\}_3 \tag{2k}$$

(out-of-plane bending moment in pipe 1)

$$\{\, M_y\,\}_1 = \{\, M_z\,\}_2 - \{\, M_z\,\}_3 \tag{2l}$$

(torsional angular velocity in pipe 1)

$$\{\, \dot{\theta}_z\,\}_1 = \{\, -\dot{\theta}_y\,\}_2 = \{\, \dot{\theta}_y\,\}_3 \tag{2m}$$

(torsional moment in pipe 1)

$$\{ M_z \}_1 = - \{ M_y \}_2 + \{ M_y \}_3 \qquad (2n)$$

where the indices 1, 2 and 3 refer to the three sides of the tee-piece. The mass and dimensions of the tee-piece are neglected, just as the forces due to changes in liquid momentum. The angles between the pipes are assumed to remain at 90 degrees.

It is noted again that for the planar systems considered herein out-of-plane motion is disregarded.

### FSI

FSI junction coupling of liquid and pipe is caused by the *axial* vibration of each leg of the L-bow or T-piece. It is modelled through the relations (1a, 2a), (1d, 2d) and (1f, 2f). Lateral and torsional vibrations do not cause FSI effects in a direct way. In lateral pipe vibrations, the liquid acts as added mass. In torsional pipe vibrations, the influence of the liquid is negligible.

### Spectra

Typical calculated frequency spectra obtained with a Dirac-pulse excitation are shown in Figure 3 for the L-shaped system and in Figure 4 for the T-shaped system.

## 4. RESULTS

The measured and calculated natural frequencies are compared.

### L-shaped system

Table 2 displays the natural frequencies up to 500 Hz of the L-shaped system. The theoretical model predicts all the observed frequencies, but the magnitudes are slightly too high up to 400 Hz. This might be attributed to the fact that the inertia of the elbow, which has a mass of 0.88 kg, has been (partly) neglected in the simulation. Tentarelli [1990; Tentarelli & Brown 2001; Brown & Tentarelli 2001] showed that a fair amount of detail of the system has to be put into FSI calculations to get highly accurate results. For example, care has to be taken in modelling lumped masses.

### T-shaped system

Table 3 displays the natural frequencies up to 250 Hz of the T-shaped system. The measured data have been taken from [Vardy *et al* 1996]. The system was excited in the direction of its axis of symmetry along the long leg, which resulted in pure axial vibration of the long pipe and hence higher natural frequencies than in the L-shaped system. The theoretical model predicts the first four natural frequencies reasonably accurate. The inertia of the T-piece has been (partly) neglected in the simulation.

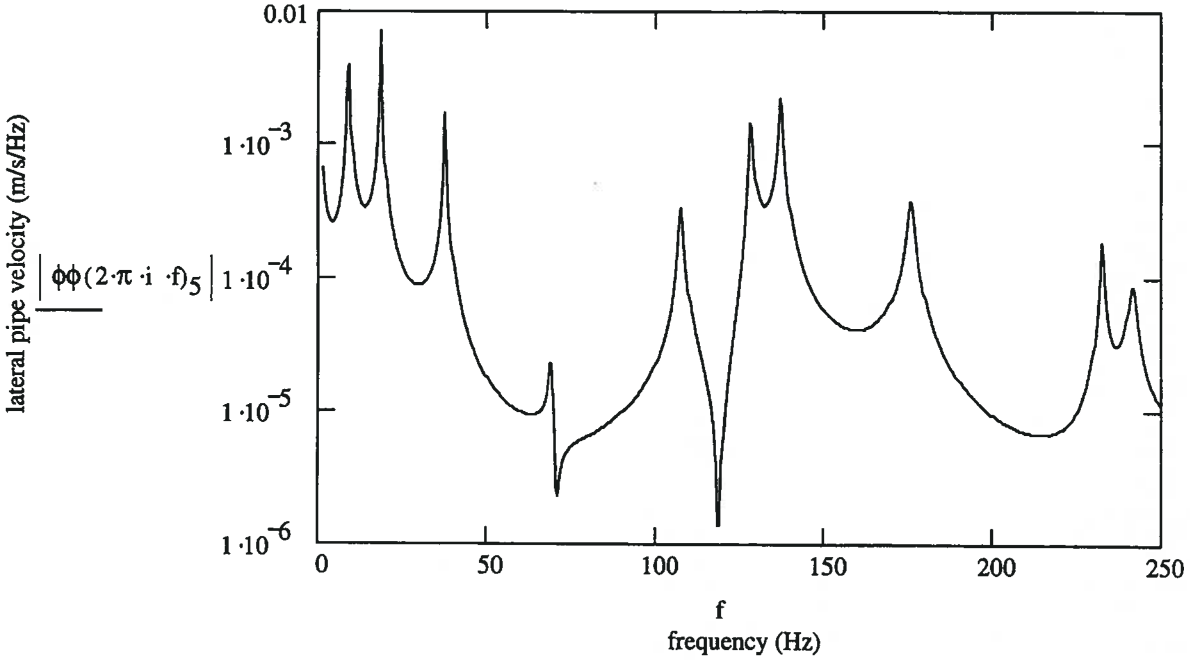

FIGURE 3. Theoretical results: L-shaped system, Dirac-pulse excitation, frequency spectrum of lateral pipe velocity at free end of long pipe.

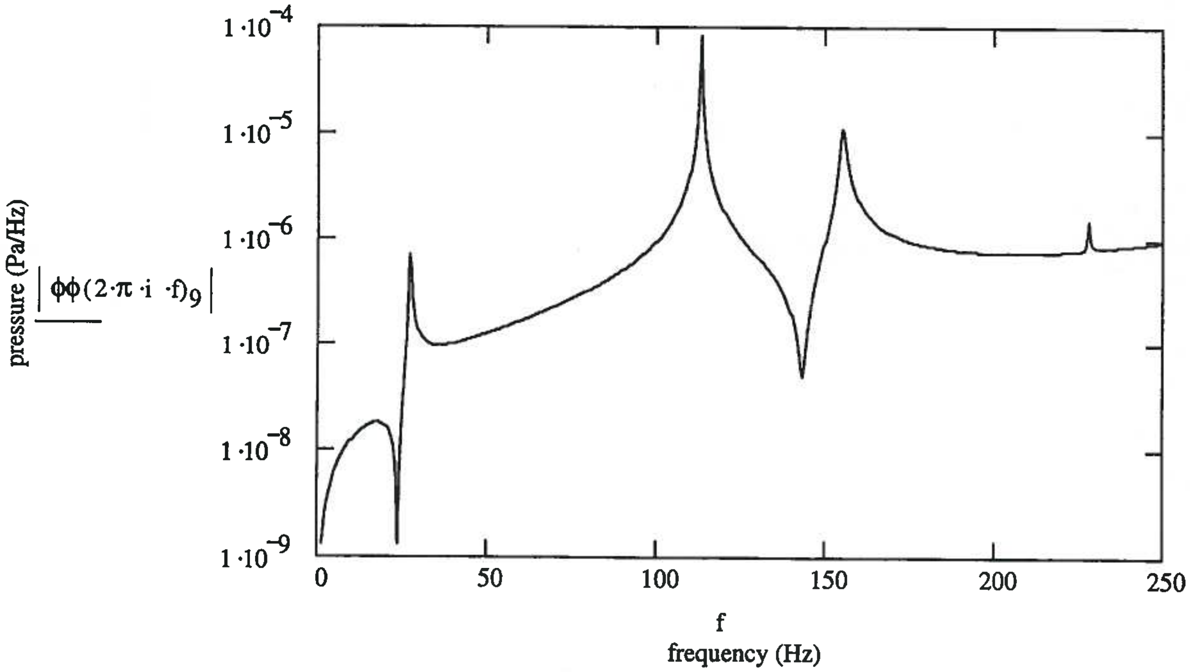

FIGURE 4. Theoretical results: T-shaped system, Dirac-pulse excitation, frequency spectrum of pressure at free end of short pipe.

| Experiment | Theory |
|---|---|
| 9 | 9 |
| 17 | 18.5 |
| 35 | 37.5 |
| 66 | 69 |
| 104 | 107.5 |
| 124 | 128 |
| 136 | 137 |
| 168 | 175.5 |
| 231 | 232 |
| 239 | 241.5 |
| 303 | 310 |
| 346 | 347.5 |
| 361 | 363 |
| 401 | 410.5 |
| 473 | 471 |
| 483 | 480 |
| 499 | 504 |

TABLE 2. Natural frequencies of L-shaped pipe system.

| Experiment | Theory |
|---|---|
| 27 | 27 |
| 112 | 113 |
| 159 | 155 |
| 226 | 228 |

TABLE 3. Natural frequencies of T-shaped pipe system.

## 5. CONCLUSIONS

The paper gives newly measured natural frequencies in an L-shaped pipe system and previously measured natural frequencies in a T-shaped system. Both systems do not suffer from a structurally "fixed" point. The measured data are consistent with theoretical predictions. The important FSI junction coupling relations for elbows and tee-pieces are explicitly given.

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