FLUID-STRUCTURE INTERACTION IN LIQUID-FILLED PIPE SYSTEMS: A REVIEW

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A review of literature on transient phenomena in liquid-filled pipe systems is presented. Waterhammer, cavitation, structural dynamics and fluid-structure interaction (FSI) are the subjects dealt with. The emphasis is on the history of FSI research in the time-domain. Waterhammer is the most probable cause for the transient vibration of liquid-filled pipe systems. When correctly modelling the liquid and pipe vibrations, FSI must be taken into account. The development of adequate mathematical models and their validation by physical experiments is surveyed.

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1. INTRODUCTION

WHEN STARTING HIS RESEARCH ON FLUID-STRUCTURE interaction (FSI) in liquid-filled pipe systems in 1986, the author found the literature survey by Wiggert (1986) very helpful. This paper is an extension of Wiggert's survey. Its aim is twofold: (i) to be a starting-point for researchers new in the field and (ii) to be a state-of-the-art record of relevant contributions to the subject. The emphasis of the present review is on transient phenomena and, consequently, time-domain analyses. FSI is presented as an extension of conventional waterhammer theory, as in Skalak's (1956) classical article. FSI, and some practical sources of excitation, are shown schematically in Figure 1.

Pipe systems experience severe dynamic forces during a waterhammer event. When these forces make the system move, significant FSI may occur, so that liquid and pipe systems cannot be treated separately in a theoretical analysis: interaction mechanisms have to be taken into account. In the majority of the analyses reviewed, the pipes are slender, thin-walled, straight, prismatic and of circular cross-section. The liquid and the pipe-wall material are assumed linearly elastic and cavitation is assumed not to occur. The theories developed are valid for long (compared to the pipe diameter) wavelength, acoustical (convective velocities neglected) phenomena. Important dimensionless parameters in FSI analyses are (i) the Poisson ratio, (ii) the ratio of pipe radius to pipe-wall thickness, (iii) the ratio of liquid mass density to pipe-wall mass density, and (iv) the ratio of liquid bulk modulus to pipe-wall Young's modulus. When the hydraulic and structural mass and elasticity, and hence the propagation speeds of pressure and stress waves, are of the same order of magnitude, FSI is likely to be of importance, provided that the transient excitation is sufficiently rapid. FSI is usually of no importance in gas-filled pipes because the mass density and elasticity (bulk) modulus of gases are negligible compared to those of solid pipes.

A classification of one-dimensional FSI models according to their basic equations, written as a hyperbolic set of first-order partial differential equations, is often made. The two-equation (one-mode) model refers to classical waterhammer theory, where the liquid pressure and velocity are the only two unknowns. The four-equation (two-mode)
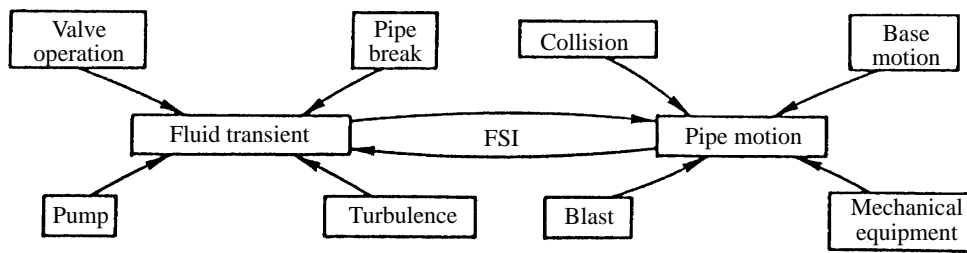
Figure 1. Sources of fluid transients and pipe motion (Wiggert 1986).

model allows for the axial motion of straight pipes; axial stress and axial pipe-wall velocity are the additional variables. The six-equation model appears when radial inertia forces are taken into account; hoop stress and radial pipe-wall velocity are the extra unknowns. The eight-equation model is valid for planar pipe systems, with the possible inclusion of curved pipes. Axial and in-plane flexural motion are considered, while radial inertia is ignored. The state-of-the-art fourteen-equation model describes axial motion (liquid and pipes), in- and out-of-plane flexure, and torsional motion of three-dimensional pipe systems.

The structure of the paper is as follows. Section 2, primarily aimed at readers new in the field, gives an explanation of the relevant basic concepts. Section 3 is a short history of waterhammer and a comprehensive review of literature on FSI. Section 4 contains conclusions.

2. BASIC CONCEPTS

This section is meant for readers not familiar with transient phenomena in pipe systems. The subjects of waterhammer, cavitation, structural dynamics and fluid-structure interaction are introduced.

2.1. WATERHAMMER

Waterhammer, also referred to as *pressure surges* or *fluid transients,* is a spectacular form of unsteady flow in liquid-filled pipe systems. It is generated by abrupt changes to steady flow conditions. These changes are mostly due to the rapid closing or opening of valves, or to the stopping or starting of pumps. Other causes of waterhammer are load rejection of turbines, seismic excitation and pipe rupture. Waterhammer involves large transient pressure variations which may damage the pipe system and its fluid machinery. In order to prevent damage, waterhammer can be suppressed and controlled by devices like: surge tanks, air chambers, flexible hoses, pump flywheels, relief valves and rupture disks. In practice, waterhammer analyses are carried out in order to judge whether these quite expensive devices are necessary and, if so, what their dimensions should be. Predicted maximum pressures determine the required strength of the pipework.

2.2. CAVITATION

Waterhammer not only leads to high pressures but also to low pressures. Low pressures involve the danger of pipe collapse, especially in the case of buried pipelines. When the pressure falls below a certain level, cavitation occurs. A distinction should be made between gaseous and vaporous cavitation. *Gaseous cavitation* occurs when the

pressure falls below the saturation pressure of the gas, so that it comes out of solution. This is a relatively slow process compared to vaporous cavitation. *Vaporous cavitation* occurs when the pressure drops to the vapour pressure. Vapour cavities will form in the liquid. When the vapour cavities appear as tiny bubbles dispersed throughout the liquid along great lengths of pipe, it is referred to as *distributed cavitation*. When the vapour cavities coalesce and form one local bubble occupying a large part of the pipe cross-section, it is referred to as *column separation*. Column separations usually occur near specific points in a pipe system such as valves, pumps, bends and high points. They occasionally occur at intermediate points when two rarefaction waves meet. The collapse of column separations is usually attended with almost instantaneous pressure rises. These may be avoided by positioning air-inlet valves at critical points in the pipe system. In general, it is good policy to prevent cavitation.

2.3. STRUCTURAL DYNAMICS

Steady-flow analyses and waterhammer analyses provide information on the liquid behaviour under operational conditions. Static pipe-stress analyses and structural-dynamics analyses give insight into the corresponding behaviour of the pipe system. Where the liquid analysis yields pressures and velocities, the structural analysis provides dynamic stresses, reaction forces and resonance frequencies. It is not unusual to perform an *uncoupled calculation.* Pressure histories, resulting from a waterhammer analysis, are used as the exciting loads in a structural-dynamics analysis. The calculation is called uncoupled since the predicted structural response does not influence the predicted liquid pressures. This procedure is dubious, since pipe motion can significantly affect dynamic pressures. Fluid-structure interaction should therefore be considered.

2.4. FLUID-STRUCTURE INTERACTION

In conventional waterhammer analyses, pipe elasticity is incorporated in the propagation speed of the pressure waves. Pipe inertia and axial pipe motion are not taken into account. This is acceptable for rigidly anchored pipe systems. For less restrained systems, fluid-structure interaction may become of importance. In that case the dynamic behaviour of the liquid and pipe system should be treated simultaneously.

Three liquid-pipe interaction mechanisms can be distinguished: friction coupling, Poisson coupling and junction coupling. *Friction coupling* represents the mutual friction between liquid and pipe. The more important *Poisson coupling* relates the pressures in the liquid to the axial (longitudinal) stresses in the pipe through the radial contraction or expansion of the pipe wall. It is named after Poisson in connection with his contraction coefficient $v,$ and it is associated with the *breathing* or *hoop mode* of the pipe. Poisson coupling leads to *precursor waves.* These are stress-wave-induced disturbances in the liquid which travel faster than, and hence ahead of, the classical waterhammer waves. Poisson coupling is illustrated in Figure 2.

Whereas friction and Poisson coupling act along the entire pipe, *junction coupling* acts as specific points in a pipe system such as unrestrained valves, bends and tees. A standard example is the vibrating elbow, which induces pressure waves in the liquid through a combined pumping (compressing) and storage (decompressing) action. Another example is the pipe bridge shown in Figure 3.

The main effects of fluid-structure interaction are problem-dependent. When compared to predictions of conventional waterhammer and uncoupled analyses,
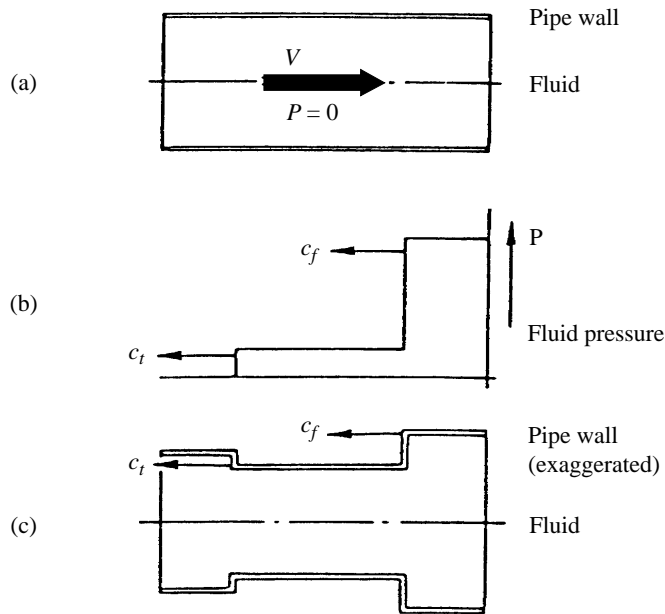
Figure 2. Poisson coupling. In (a) fluid is flowing at velocity V and reference pressure $P = 0$. In (b) the fluid is stopped instantaneously, resulting in a pressure rise which propagates at speed c_f through the fluid. The pressure rise is accompanied by a radial expansion of the pipe wall as shown in (c). Due to this radial expansion, the pipe shortens behind and elongates in front of the pressure rise. The elongation reveals itself as an axial stress wave. It propagates at speed c_t through the pipe and causes a radial contraction of the pipe wall as in (c). The radial contraction causes a secondary pressure rise in the fluid as shown in (b). This secondary pressure rise is often referred to as a precursor wave since it propagates at speed c_t, which is generally higher than the propagation speed c_f of the primary pressure rise (Tijsseling & Lavooij 1990).

predictions including fluid-structure interaction may lead to: higher or lower extreme pressures and stresses, changes in the natural frequencies of the system, and more damping and dispersion in the pressure and stress histories.

In practice, pipe systems are never entirely rigid. Indeed, unduly rigid systems may experience extremely high stresses due to temperature effects. The problem is to judge when fluid-structure interaction is of importance. In Lavooij & Tijsseling (1991) a
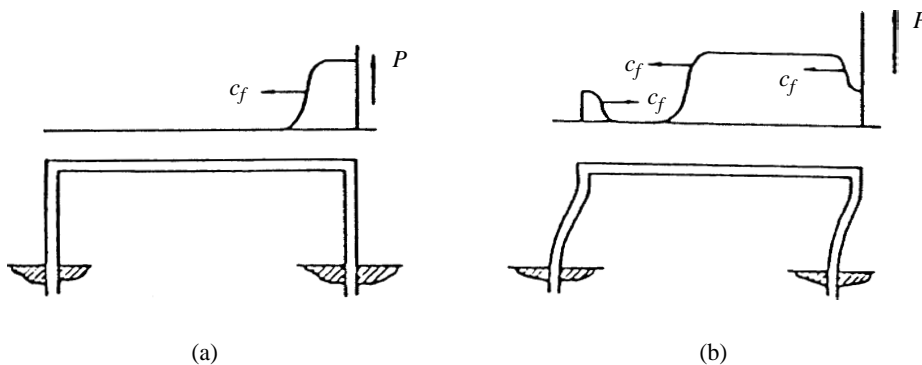

(a)   (b)

Figure 3. Junction coupling. When a pressure wave has passed the elbow on the right of (a), the net pressure difference between the two elbows causes the pipe bridge to move as in (b). Due to the motion the pressure drops at the right and rises at the left elbow, as shown, simplified, in (b). The motion of the pipe bridge induces pressure waves in the fluid, which in return influence the motion of the bridge (Tijsseling & Lavooij 1990).

provisional guideline is proposed to answer this question. The guideline is based on the characteristic time-scales of the system under consideration. It says that fluid-structure interaction is important when the time-scale of the structural behaviour is: (i) smaller than the time-scale of the liquid behaviour and (ii) larger than the time-scale of the excitation. The eigenperiods of the pipe system, the main periods of the waterhammer waves and, for instance, the effective closure time of a valve, provide the relevant time-scales. Uffer (1993) emphasizes the importance of pressure rise times.

Calculations with fluid-structure interaction are necessary in situations with high safety requirements, mostly encountered in the nuclear and chemical industry. They may also be useful in post-accident analyses (De Almeida & Pinto 1986; Wang *et al.* 1989; Obradović 1990b), in trouble shooting and in the development of design rules.

3. LITERATURE REVIEW

The main contributions to the development of the classical theory of waterhammer are briefly reviewed, and the standard textbooks on this subject are listed. The broad field of cavitation, without FSI, is not reviewed here. This has been done by Simpson (1986) and Tijsseling (1993). Most attention is directed to time-domain FSI work. A comprehensive account of the literature is given. The combination of FSI and cavitation is treated in a separate subsection.

3.1. WATERHAMMER

The first work with respect to waterhammer is attributed to Ménabréa (1858, 1862). Although several other publications appeared in the second half of the 19th century, the present waterhammer theory is based on the classical investigations of Joukowsky (1898) and Allievi (1902). Joukowsky conducted extensive measurements in the Moscow water distribution system. In his theoretical treatment, he derived the formula that bears his name

$$\Delta P = \rho_f c_f \Delta V. \tag{1}$$

This formula was also derived by Frizell (1898) and earlier, in a more general context, by Rankine (1870). It relates pressure changes, ΔP, to velocity changes, ΔV, by the constant factor $\rho_f c_f$, with ρ_f the mass density of the fluid and c_f the velocity of sound in the fluid. Joukowsky used the sound velocity according to Korteweg (1878), which takes into account both the compressibility of the fluid and the elasticity of the pipe walls. Allievi gives an excellent mathematical treatment of the waterhammer equations in Allievi (1913). His method of solution was used, until the more practical *graphical method* was developed. This method is connected with the names of Schnyder (1929) and Bergeron (1935), although there were a few predecessors. With the introduction of digital computers in the early 1960s, waterhammer calculations evolved to their present form. The method of characteristics (MOC) has become the standard numerical approach for solving the *waterhammer equations* (*two-equation model*):

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial P}{\partial z} = 0, \tag{2}$$

$$\frac{\partial V}{\partial z} + \frac{1}{\rho_f c_f^2} \frac{\partial P}{\partial t} = 0, \tag{3}$$

where t is the time and z is the distance along the pipe. Friction and gravity terms are

neglected in these equations. All fundamentals can be found in the classical textbook of Streeter & Wylie (1967). Other good textbooks on the subject are those of Bergeron (1950), Gandenberger (1950), Rich (1951), Parmakian (1955), Zielke (1974), Jaeger (1977), Fox (1977, 1989), Wylie & Streeter (1978, 1993), Watters (1979), Chaudhry (1979), Chaudhry & Yevjevich (1981), Tullis (1989) and De Almeida & Koelle (1992). Historical reviews are given by Paynter (1961), Wood (1970), Martin (1973), Anderson (1976) and Thorley (1976). Jaeger (1977) is very precise in his footnotes, where he refers to the original researchers and their work.

3.2. FLUID-STRUCTURE INTERACTION

3.2.1. *Early developments*

In the 19th century, studies of standing waves in musical instruments and pulsatile flows in blood vessels demanded an accurate determination of the velocity of sound in fluids. The sound velocity in unconfined fluids was already known to be

$$c_0 = \sqrt{K/\rho_f}, \tag{4}$$

where K is the bulk modulus of the fluid. For liquids contained in a tube, the sound velocity was found to be much lower. Von Helmholtz (1848) correctly attributed this effect to the elasticity of the tube walls. For incompressible fluids in elastic tubes, like rubber hoses and blood vessels, Young (1808), Weber (1866), Résal (1876) and Moens (1878) came to the formula

$$c_1 = \sqrt{\frac{Ee}{\rho_f D}}, \tag{5}$$

where E is Young's modulus of the wall material, e is the wall thickness and D is the internal tube diameter. For compressible fluids in elastic tubes Korteweg (1878) derived

$$\frac{1}{c_f^2} = \frac{1}{c_0^2} + \frac{1}{c_1^2}, \tag{6}$$

which is equivalent to

$$c_f = \sqrt{\frac{K}{\rho_f} \left(1 + \psi \frac{DK}{eE}\right)^{-1}}, \tag{7}$$

where the coefficient $\psi = 1$. The coefficient ψ is explained in subsection 3.2.3.

Equation (7) gives the pressure-wave speed, c_f, used in many standard waterhammer calculations. For compressible fluids in entirely rigid tubes, $E \gg K$, c_f equals c_0, whereas for incompressible fluids in elastic tubes, $K \gg E$, c_f equals c_1.

In deriving formula (7) Korteweg (1878) considers the tube as a series of massless rings expanding and contracting in accordance with the internal fluid pressure, P. The hoop stress, σ_ϕ, and the radial displacement, u_r, of a ring are given by

$$\sigma_\phi = \frac{R}{e} P, \qquad u_r = \frac{R^2}{eE} P, \tag{8,9}$$

respectively, where R is the inner radius of the tube. The tube walls follow the liquid;

there is no question of fluid-structure *inter*action. Axial stresses in the tube wall, σ_z, and tube wall inertia are neglected; i.e.

$$\sigma_z = 0, \qquad \rho_t = 0, \tag{10,11}$$

where ρ_t is the mass density of the tube wall material.

Korteweg (1878) indicated that his theory is valid for long (with respect to pipe diameter) wavelengths. He pointed out that Poisson ratio effects (see Section 2.4 for Poisson coupling) occur when the axial stresses in the tube wall are not neglected. When, in addition, *axial* inertia forces in the tube wall are taken into account, axial stress waves will propagate along the tube. Korteweg, however, investigated the influence of *radial* inertia forces. He showed that for short wavelengths, the radial inertia of both the fluid and the tube wall are of importance, thereby leading to wave speeds varying with the wavelength. Gromeka (1883) took tube wall inertia into account when he considered incompressible liquids in elastic tubes. He gave a bi-quadratic equation from which two wave speeds follow: one for the pressure waves in the fluid and one for the axial stress waves in the tube wall. Lamb (1898) was very complete in his treatment of the combined axial and radial vibrations of a fluid-filled tube. Poisson coupling is included in his work. He distinguished three classes of vibrations, *viz.* (i) the pressure waves in the fluid as modified by the yielding of the tube, (ii) the axial vibrations of the tube wall as modified (very slightly) by the presence of the fluid, and (iii) the radial vibrations of the system. Lamb derived a dispersion equation which relates phase velocities to wavelengths. For long wavelengths the pressure waves and the axial stress waves are predominant, with the propagation speeds close to c_f and c_t, respectively, where

$$c_t = \sqrt{E/\rho_t} \tag{12}$$

and c_f is given by equation (7). The radial vibrations are important only for short wavelengths.

Boulanger (1913) gave an extensive survey of the developments in the 19th century.

3.2.2. *Wave propagation modes*

Lamb's (1898) work has been extended by numerous researchers dealing with the problem of wave propagation in fluid-filled cylinders. The general approach is to apply Fourier analyses or integral transforms to the basic equations. A dispersion equation is then derived, from which the natural modes of wave propagation follow.

Skalak (1956) presented an excellent piece of work which can be seen as the theoretical basis for FSI in straight liquid-filled pipes. He extended Lamb's (1898) work by including bending stiffness and rotatory inertia in his axially symmetric tube model. From his dispersion equation he found an infinite number of wave propagation modes. Only the two lowest modes have a finite phase velocity as the wavelength approaches infinity and the frequency approaches zero. The lowest mode corresponds to the pressure waves in the fluid, the second lowest to the axial stress waves in the tube wall. The phenomenon of *precursor waves* (see Section 2.4) was described with the aid of clear pictures. Simplified equations were given which permit precursor-type solutions without dispersion effects. These are the equations presently used by many researchers.

Lin & Morgan (1956a,b) delivered work similar to Skalak's. Beside bending stiffness and rotatory inertia, they took into account transverse shear deformation in their tube model. These matters are of importance only in the high frequency range.

Hermann & Mirsky (1956) considered an empty tube and found basic equations that

differ slightly from those of Lin & Morgan (1956a). They surveyed the various models that are in use for axially symmetric motions of thin-walled cylinders. Equations governing the non-axially symmetric motion of cylinders were given in Mirsky & Herrmann (1957), whereas in Mirsky & Herrmann (1958) thick-walled cylinders were considered.

Spillers (1965) and Tang (1965) applied the method of characteristics to, respectively, Herrmann & Mirsky's and Lin & Morgan's equations.

King & Frederick (1968) used Lin & Morgan's equations in a completion of Skalak's work. By applying a Hankel transformation to the two-dimensional fluid equations, they obtained an infinite system of one-dimensional wave equations for the fluid. A finite set from this system, along with three one-dimensional wave equations for the tube, was transformed into ordinary differential equations by means of the method of characteristics. Finally, a numerical integration procedure led to the early-time waterhammer response solutions which the authors were looking for.

Thorley (1969) followed Skalak's approach, but neglected transverse shear deformation, rotatory inertia and, partly, bending stiffness. He performed physical experiments in steel, aluminium alloy, and polythene pipes of 14 m length, 0·05 m diameter and 0·005 m wall thickness. Thorley was the first who actually observed *precursor waves* (Figure 4).

DeArmond & Rouleau (1972) extended Lin & Morgan's work by considering a *viscous* liquid in an elastic tube. They concentrated on the two lowest modes of wave propagation. Rubinow & Keller (1978) studied, for infinitely many modes, a viscous liquid in a viscoelastic tube. Kuiken (1984a,b,c,d) came with the most complete model, encompassing viscous liquids or gases, orthotropic viscoelastic tubes, pre-stressed and surrounded by other materials, and thermodynamic effects. Möser *et al.* (1986) investigated those modes that produce audible sound. They presented results of accurate measurements.

The advanced study of wave propagation in fluid-filled tubes is nowadays still of importance, in particular with respect to pulsatile flows in mammalian arteries. For the description of pressure waves in pipe systems, one-mode or two-mode solutions are sufficient. One-mode solutions are found with classical waterhammer theory, two-mode solutions are calculated when Poisson coupling is taken into account.

3.2.3. *Wave propagation speeds*

Halliwell (1963) gave, within the context of classical waterhammer theory, formulae for the pressure wave speed in both thin- and thick-walled pipes. He also discussed the disparity in wave-speed formulae found in textbooks. This disparity is due to the way pipe support conditions are taken into account. In theoretical analyses three support conditions are standard: (i) pipe anchored with expansion joints throughout its length, (ii) pipe anchored throughout against axial motion, and (iii) pipe anchored at its upstream end only. The correction factor, ψ, in the wave-speed equation (7) accounts for the different support conditions. Support condition (i) corresponds to the situation described by Korteweg (1878), in which the axial stresses are neglected ($\psi = 1$). In support condition (ii) the axial displacements are negligible, leading to $\psi = 1 - \nu^2$. In support condition (iii) the axial stress in the pipe wall is assumed to be proportional to the fluid pressure acting on an unrestrained closed valve downstream, so that $\psi = 1 - \frac{1}{2}\nu$. Correct treatises of the subject can be found in Wylie & Streeter (1978)
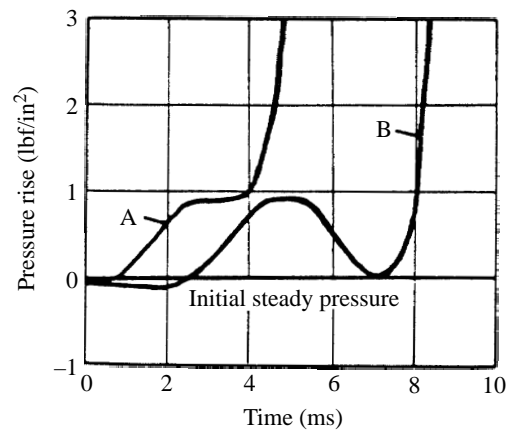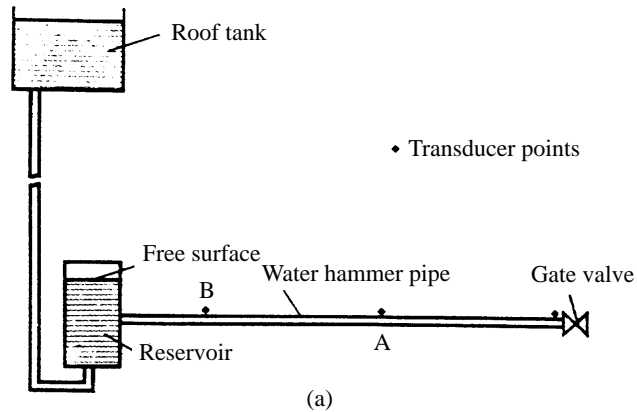
Figure 4. (a) Schematic diagram of waterhammer apparatus (Thorley 1969). (b) An oscilloscope record (redrawn) of the precursor wave in the aluminium alloy pipe at the points A and B, respectively, 6·4 m and 10·7 m from the valve (Thorley 1969). The precursor wave is identified as the pressure rise occurring before the arrival of the main waterhammer wave at A after 4·8 ms and at B after 8·0 ms.

and Schwarz (1978). In deriving the values of ψ, pipe wall inertia is neglected, so that quasi-static conditions prevail.

The support conditions (i) and (iii) allow for axial pipe motion. For these cases it has been assumed that the axial stress in the entire pipe is permanently equal to its value at the anchors or to its value at the valve. A more thorough treatise considers axial stress *waves* by taking into account axial inertia forces in the pipe wall. This approach leads to a unique wave speed in a single pipe supported at its ends, which is physically correct since the wave speed must not depend on the end conditions: a disturbance generated at the middle of the pipe, and travelling at finite speed, does not "know" in advance what situation it will meet at the pipe ends. In this respect it is noted that ignoring axial pipe wall inertia leads to axial stress waves theoretically travelling at infinitely high speed, so that the end conditions are permanently "felt" along the entire pipe.
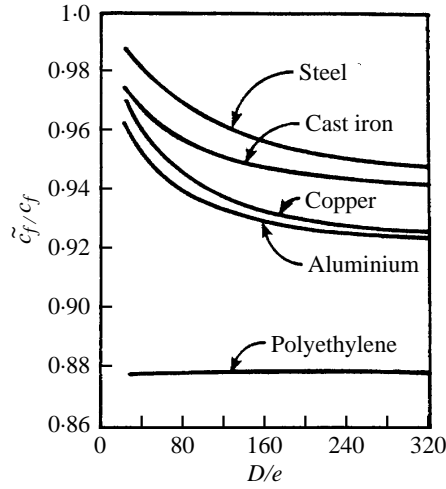
Figure 5. Variation of the liquid wave speed ratio (improved/classical models) with D/e for different pipe materials and water (Stuckenbruck *et al.* 1985).

Also, the axial stress wave speed is influenced by axial inertia forces in the liquid. The Poisson coupled values $\tilde{c}_f$ and $\tilde{c}_t$ of the wave speeds are

$$\tilde{c}_f = [\tfrac{1}{2}\{q^2 - (q^4 - 4c_f^2 c_t^2)^{1/2}\}]^{1/2}, \tag{13}$$

$$\tilde{c}_t = [\tfrac{1}{2}\{q^2 + (q^4 - 4c_f^2 c_t^2))^{1/2}\}]^{1/2}, \tag{14}$$

where $q^2 = c_f^2 + c_t^2 + v^2(\rho_f/\rho_t)(D/e)c_f^2$; c_f, with $\psi = 1 - v^2$, and c_t are given by (7) and (12), respectively (Bürmann 1975; Budny *et al.* 1991). The ratio $\tilde{c}_f/c_f$ for water-filled pipes of different wall materials is shown as a function of D/e in Figure 5.

3.2.4. *Junction coupling*

In classical waterhammer theory, the only influence of the pipe support conditions is in the correction factor ψ in the pressure-wave speed expression (7). When the support conditions allow for pipe motion, fluid-structure interaction may occur. The most important interaction mechanism is junction coupling (see Section 2.4). In general, a pipe system consists of straight sections of pipe, connected by elbows, tees and diameter changes, and is terminated by reservoirs, pumps and valves. Here the connecting and terminal points are referred to as *junctions.* Hydraulic equipment other than those mentioned above may also be located at junctions. When a junction is able to move in the axial pipe direction, which is the direction of the pressure waves, mutual forces between the fluid and the pipe system may cause a dynamic interaction, which is known as *junction coupling.*

Regetz (1960) investigated pressure and velocity fluctuations in a straight pipe filled with rocket fuel. His experimental apparatus allowed for axial pipe motion. Pipe velocities recorded at the unrestrained pipe end were incorporated in a standard waterhammer analysis in the frequency domain. It was shown that pipe motion has a definite effect on the fluid behaviour.

Blade *et al.* (1962) extended Regetz's work; a flexibly supported elbow was included

in the pipe system. Axial pipe motion was simulated by means of a spring-mass system. Junction coupling was modelled by relating pressures and velocities in the fluid to stresses and velocities in the pipe according to the local force equilibrium and continuity.

D'Souza & Oldenburger (1964) let pressure waves in the fluid interact with stress waves in the pipe wall by way of junction coupling at an unrestrained pipe end. They utilized Laplace transformations in a frequency response analysis of their basic equations. Excellent agreement was found between theory and experiments conducted in a straight pipe filled with hydraulic oil.

Holmboe & Rouleau (1967) had problems with unwanted FSI effects in their experimental apparatus and decided to embed their spiral tube in concrete.

Swaffield (1968–69) examined the influence of elbows on passing pressure waves by means of a comprehensive series of tests. The influence of pipe restraint was part of his work. Unfortunately, his results were invalidated by the fact that some supports in his experiments were not sufficiently rigid (Wilkinson 1980; p. 197).

Davidson & Smith (1969) modelled curved pipes in a Timoshenko-beam-like manner including liquid-pipe interaction. The model was validated against results of frequency response tests in an oil-filled single-elbow pipe system. The work was extended to non-planar multi-elbow configurations by Davidson & Samsury (1972).

Wood (1968, 1969) gave a clear explanation of junction coupling in terms of moving hydraulic discontinuities. He presented a time-domain analysis in which the structure was represented by a spring-mass system where the fluid pressure was the driving force. His experimental apparatus consisted of a rigidly supported straight pipe terminated by a spring-mass device. In Wood (1968) the liquid was subjected to periodic disturbances, whereas in Wood (1969) the system was excited by rapid valve-closure. In the latter paper it was shown that pressure rises may significantly exceed Joukowsky's value, equation (1), when axial pipe motion occurs. An interesting parameter variation study was given which links up with the guideline described in Section 2.4. In Wood & Chao (1971) the work was extended with elbows and branches. A valuable series of tests was carried out on 30°, 60°, 90°, 120° and 150° mitre bends and on a 90°–90° T-junction. No attempt was made to model the structure; measured junction velocities were used as input to the analysis. It was shown that rigidly supported elbows had a negligible influence on pressure waves, whereas unrestrained elbows affected them considerably. Jones & Wood (1972) gave an analytically derived expression for the junction-coupling-induced pressure oscillations around Joukowsky's value in the case of rapid valve-closure downstream in a single pipe. The pipe was regarded as a spring-mass system. Calculated results were compared with measurements in an unrestrained vertical pipe.

Ellis (1980) considered junction coupling at a check-valve, a T-branch and a 90° bend in an analysis of a practical system. Pressure waves and axial stress waves were modelled; flexural motion was accounted for by a spring-mass system. The method of characteristics was employed to solve the basic equations.

3.2.5. *Junction and Poisson coupling*

In the work reviewed in the previous subsection the influence of axial junction motion on pressure waves was investigated. The axial pipe motion was represented by measured data or by a simple spring-mass system. Only D'Souza & Oldenburger (1964) and Ellis (1980) modelled axial stress waves in the pipe wall. However, they did not take into account *Poisson coupling,* that is the FSI mechanism relating internal fluid pressures to pipe wall axial stresses. The work of researchers including this mechanism

in their model is discussed in this subsection. In fact, they utilize the two-mode model introduced in subsection 3.2.2.

Research at the University of Karlsruhe, Germany

Bürmann (1974a,b, 1975, 1980b) gave a rigorous mathematical treatment of axial wave propagation in liquid-filled co-axial cylinders, including Poisson coupling. The basic equations were solved by the method of characteristics (MOC). Non-axial effects were studied in Bürmann *et al.* (1979a, 1983, 1987c). Bürmann (1980a, 1983) compared the responses of three different tube models to waterhammer induced by the rapid closure of a rigidly supported valve. The three tube models correspond to the shell and membrane theories such as given by Hermann & Mirsky (1956), and to the simple one employed in classical waterhammer theory.

Bürmann concluded that shell theory is necessary for very short pipes and for obtaining accurate solutions in the vicinity of very steep wave fronts. The membrane theory in combination with extended waterhammer equations allows for precursor waves and is sufficient for most practical purposes. The classical waterhammer theory fails to predict pipe motion. A *four-equation model* of the type

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f}\frac{\partial P}{\partial z} = 0, \tag{15}$$

$$\frac{\partial V}{\partial z} + \left(\frac{1}{K} + \frac{2R}{Ee}\right)\frac{\partial P}{\partial t} - \frac{2\nu}{E}\frac{\partial \sigma_z}{\partial t} = 0, \tag{16}$$

$$\frac{\partial \dot{u}_z}{\partial t} - \frac{1}{\rho_t}\frac{\partial \sigma_z}{\partial z} = 0, \tag{17}$$

$$\frac{\partial \dot{u}_z}{\partial z} - \frac{1}{E}\frac{\partial \sigma_z}{\partial t} + \frac{\nu R}{Ee}\frac{\partial P}{\partial t} = 0, \tag{18}$$

where $\dot{u}_z$ is the axial velocity of the pipe wall, was proposed to describe the coupled axial motion of pipe and liquid (Bürmann *et al.* 1979b, 1980; Thielen & Bürmann 1980). In principle, this is the simplified model of Skalak [1956, Eqns (65–68)]. The hoop stress equation (8) is still valid, but the radial displacement equation (9) is replaced by

$$u_r = \frac{R^2}{Ee}P - \frac{\nu R}{E}\sigma_z. \tag{19}$$

Bürmann, and his group, carried out an impressive series of field measurements in order to validate their theoretical work. In Bürmann (1979) and Bürmann & Thielen (1988a) measurements were presented on the filling pipe of a subterranean salt cavern (Figure 6). The salt cavern was used for oil storage and the filling pipe, which was vertically placed within a second pipe, had a length of more than 1 km. The pipe was minimally restrained and showed motion in axial and lateral direction when it was excited by waterhammer due to pump trip. In Bürmann *et al.* (1985, 1986b, 1987a) a water-main bridge across the river Neckar was studied. Coupled motion of liquid and pipe was generated by rapid valve opening. At one end, the pipe was lifted from its support to enhance axial motion. Bürmann *et al.* (1986a, 1987b) and Bürmann & Thielen (1988b) investigated a loading line, used for liquid transport between storage tanks and ships. A detailed description of the measurements and the corresponding simulations was given for all three cases (salt cavern, pipe bridge and loading line). It
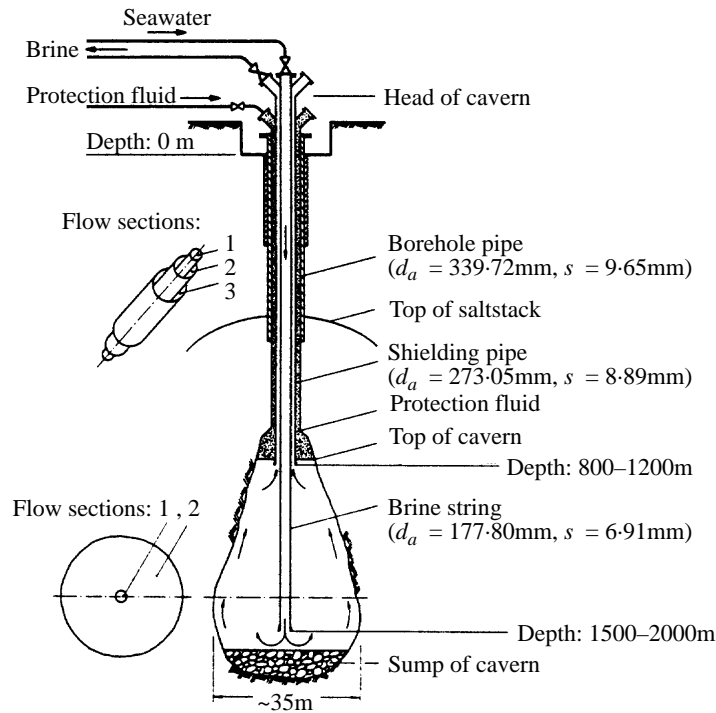
Figure 6. Cavern being de-brined under direct circulation (Bürmann 1975).

is encouraging to see that satisfactory agreement was found between theory and field data, although, in the simulations, measured data were used as input.

Research at other institutes

Walker & Phillips (1977) presented a theoretical study of the propagation of short-duration pressure pulses in a straight elastic pipe, including Poisson and junction coupling. Radial inertia forces in the pipe wall, with added fluid mass, were taken into account by two additional equations:

$$\left(\rho_t e + \frac{1}{2}\rho_f R\right)\frac{\partial \dot{u}_r}{\partial t} + \frac{e}{R}\sigma_\phi - P = 0, \tag{20}$$

$$(1-\nu^2)\frac{\partial \sigma_\phi}{\partial t} - E\frac{\dot{u}_r}{R} - \nu E\frac{\partial \dot{u}_z}{\partial z} = 0, \tag{21}$$

leading to a *six-equation model*. It is noted that Kellner & Schönfelder (1982) proposed an added fluid mass term $\frac{1}{3}\rho_f R$. The equations were solved by the MOC, but the procedure was not described in detail. In this respect some doubt may exist about the distinction between numerical and physical dispersion, not discussed by the authors. For the case that pressure and axial stress waves travel at the same speed, an analytical solution was given. Valentin, Phillips & Walker (1979) presented an eight-equation model for a liquid-filled curved pipe. Poisson coupling was included, but radial inertia was neglected. A dispersion equation was derived and elbow reflection and transmission coefficients were calculated. Hu & Phillips (1981) solved the eight-equation model by the MOC and showed some experimental results. Joung & Shin (1985) extended

Walker & Phillips' (1977) axially symmetric model with terms for transverse shear deformation, rotatory inertia and bending stiffness. They solved their nine-equation model by the MOC.

Williams (1977) presented valuable experimental results obtained in steel, ABS and PVC pipes. The effect of pipe motion, brought about by axial stress waves, was clearly visible in the recorded pressures. The observed damping of pressures was attributed to FSI. Flexural pipe motion and interaction at bends were discussed. Williams' correct mathematical analysis was in terms of jump conditions across wave fronts.

Krause *et al.* (1977) studied the propagation of axial stress and pressure pulses in water-filled aluminium and acrylic plastic tubes. Their experimental apparatus consisted of a closed tube supported partly by strings and partly by blocks. Short duration pulses were generated by firing steel spheres onto the tube ends. Measured wave propagation speeds were in good agreement with those predicted by Skalak's (1956) simplified model. The work was extended, both experimentally and theoretically, by Barez *et al.* (1979). Their experiments were executed on a tube supported by blocks and filled with various liquids, initially either at rest or flowing. Skalak's simplified theory was extended with radial tube inertia and a viscoelastic Young's modulus, yielding a six-equation model similar to that of Walker & Phillips (1977). A Laplace transformation gave solutions which were inverted numerically. The computed propagations of single pulses were in good agreement with the observed ones. Adachi *et al.* (1991) applied the same solution technique to the full shell equations and compared the results of classical, uncoupled and coupled calculations. They came to the same conclusion as Bürmann (1980a, 1983): shell theory is only necessary for very short pipes and for accurate solutions close to wave fronts. Uncoupled calculations led to erroneous results.

Schwarz (1978) performed an extensive numerical study on coupled axial liquid and pipe motion in a single straight pipe. His approach was similar to that of Walker & Phillips (1977). He started with a six-equation model and solved it with the MOC. By neglecting the relatively unimportant radial inertia terms he came to a four-equation model, which was subsequently solved by a finite-difference method. Calculations with Poisson and junction coupling were compared with calculations without coupling.

Wilkinson (1978) presented a complete work formulated in the frequency domain. In addition to pressure and axial stress waves, he considered flexural and torsional stress waves, yielding a fourteen-equation model. The flexural waves were in two directions, so that five wave families were distinguished. The wave families were coupled at the various junctions described by the author. Poisson coupling was not included in the model, but this may be done by incorporating the work of Wiggert *et al.* (1987b) or Kuiken (1988). Lesmez (1989), Tentarelli (1990) and De Jong (1994) also included Poisson coupling in their thorough frequency-domain analyses; they gave impressive experimental validation of their theoretical models. Wilkinson (1980) formulated the problem in the time domain. Reflection and transmission coefficients for various moving junctions were given. In Wilkinson & Curtis (1980) a detailed analysis of the impact of a liquid column with the closed end of a straight pipe was given and validated against experimental results. Poisson coupling and plasticity were part of the work. Solutions were found by tracing wave fronts and applying jump conditions as Williams (1977) did. Since one incident wave leads to two reflected waves, the method is quite laborious. Edwards & Please (1988) proposed an original alternative method to overcome this problem.

Research at Michigan State University, USA

Wiggert (fluid mechanics) and Hatfield (structural mechanics) at Michigan State University joined their knowledge in order to tackle the problem of FSI in liquid-filled piping systems. Together with Otwell, Lesmez, Budny and Stuckenbruck, they have worked on the subject for more than 10 years. Many valuable publications have resulted. Their work was in both the frequency and time domain. Two different approaches were followed: the *component-synthesis method* and the *MOC method.* These terms refer to the numerical treatment of the equations for the structure. The former method is associated with natural modes of vibration, whereas the latter corresponds to wave propagations. Experimental validation tests completed the investigations. In Hatfield *et al.* (1982a) the component-synthesis method was introduced for application in the frequency domain. In this method, the structural motion was represented by a limited number of natural modes of vibration, which were determined with a commercially available finite-element computer code. Fluid-structure interaction was introduced by means of junction coupling. The method was validated against the experiment of Blade *et al.* (1962) and, in Hatfield *et al.* (1982b, 1983), against the experiments of Davidson & Smith (1969) and Davidson & Samsury (1972). In principle, time-domain solutions can be obtained from frequency-domain solutions by means of inverse Fourier transformations. In practice, however, this approach may lead to serious difficulties. In particular, when the transient response to impact loads is studied, the approach is impractical. Hatfield & Wiggert (1983) came to that conclusion.

As a result, they developed a time-domain implementation of the component-synthesis method, in which the standard waterhammer procedure was coupled to a modal representation of the structural motion (Wiggert & Hatfield 1983). Poisson coupling was not included in this work, which was an extension of Otwell's (1982) work. In Wiggert *et al.* (1983) the previous investigations were summarized and results of laboratory tests were presented in which the pressures, as a consequence of elbow motion, exceeded Joukowsky's classical value, equation (1). The tests, performed with copper pipes of 0·025 m diameter, were successfully simulated in Otwell (1984) and Wiggert *et al.* (1985a). A four-equation model, including Poisson coupling, was employed for axial motion; lumped stiffness accounted for flexural motion. The MOC method was applied to get numerical solutions. Wiggert *et al.* (1985b, 1986, 1987a) solved a fourteen-equation model describing the axial, flexural and torsional motions of a liquid-filled pipe system, by means of the MOC method. Timoshenko beam equations for flexural motion permit MOC solutions (Leonard & Budiansky 1954):

$$\frac{\partial \dot{u}_y}{\partial t} + \frac{1}{\rho_t A_t + \rho_f A_f} \frac{\partial Q_y}{\partial z} = 0, \tag{22}$$

$$\frac{\partial \dot{u}_y}{\partial z} + \frac{1}{\kappa^2 G A_t} \frac{\partial Q_y}{\partial t} = -\dot{\theta}_x, \tag{23}$$

$$\frac{\partial \dot{\theta}_x}{\partial t} + \frac{1}{\rho_t I_t} \frac{\partial M_x}{\partial z} = \frac{1}{\rho_t I_t} Q_y, \tag{24}$$

$$\frac{\partial \dot{\theta}_x}{\partial z} + \frac{1}{E I_t} \frac{\partial M_x}{\partial t} = 0, \tag{25}$$

and torsional pipe motion is unaffected by the liquid

$$\frac{\partial \dot{\theta}_z}{\partial t} - \frac{1}{\rho_t J_t} \frac{\partial M_z}{\partial z} = 0, \tag{26}$$

$$\frac{\partial \dot{\theta}_z}{\partial z} - \frac{1}{G J_t} \frac{\partial M_z}{\partial t} = 0, \tag{27}$$

where A is the cross-sectional area, G the shear modulus, I the second moment of area, J the polar second moment of area, M the bending or torsional moment, Q the lateral shear force, $\dot{u}$ the pipe velocity, $\dot{\theta}$ the angular pipe velocity and κ^2 the shear coefficient; the subscripts x and y denote perpendicular lateral directions, and the subscripts f and t refer to fluid and tube, respectively. Radial inertia effects are neglected in the *fourteen-equation model* which comprises the axial equations (15)–(18), two sets of lateral equations (22)–(25) and the torsional equations (26)–(27). The fourteen-equation model was treated in the frequency domain by Wiggert *et al.* (1987b) in a way similar to that of Wilkinson (1978), except that Poisson coupling was taken into account. The numerical results were validated against the experimental data of Davidson & Smith (1969), and against new experimental data on a U-bend (Lesmez 1989; Lesmez *et al.* 1990). In Wiggert (1986) a brief review of literature is given. The frequency response of a straight pipe to seismic ground motion was examined by Hatfield & Wiggert (1987, 1990). Stuckenbruck & Wiggert (1986) studied the axial motion of highly flexible tubes with Poisson ratios nearly equal to 0·5 and Rachid & Stuckenbruck (1989) extended the four-equation model to account for viscoelastic effects. Structural damping, which is of less importance for early-time solutions, was investigated theoretically and experimentally by Budny (1988) and Budny *et al.* (1989, 1990, 1991). Structural damping and associated energy losses were also the subject of an investigation by Jelev (1989). In later work, Wiggert directed his attention to the important problem of slug flow. The impact of a single liquid slug onto a single elbow was studied experimentally and numerically by Bozkus & Wiggert (1991, 1992) and Yang & Wiggert (1993). FSI effects were not considered.

Research at the University of Dundee, U.K.

Vardy and Fan, at the University of Dundee, started research on fluid–structure interaction in 1984. Their main contribution to the development of FSI knowledge is in their accurate measurements of axial and flexural wave propagations in freely suspended pipe systems. Transient vibration of their 0·05 m bore steel pipes was generated by the external impact of a solid rod. Their apparatus is less complicated than the conventional reservoir-pipe-valve system since an initial steady-state pressure gradient is absent, valve-closure characteristics are not needed and the influence of pipe supports is negligible. Friction and gravity effects are unimportant due to the time-scale (milliseconds) of the experiment. The experiment isolates the effects of fluid-structure interaction exceptionally clearly. In their theoretical work, they followed Wiggert *et al.* (1987a). First experimental results were presented in Vardy & Fan (1986) and were compared with predictions from Wilkinson's theories (Wilkinson 1980; Wilkinson & Curtis 1980). Comparison with results from Wiggert's approach followed in Vardy & Fan (1987), together with some theoretical considerations. Fan (1989) and Vardy & Fan (1989) described further measurements in a straight pipe. Excellent agreement was found between theory and experiment for both axial and flexural wave

propagations. Results on pipe systems with an elbow and with a T-piece were presented in Fan & Vardy (1994) and Vardy & Fan (1996).

Research at Delft Hydraulics, The Netherlands

The FSI work carried out by a research group in Delft is strongly based upon Wiggert and Hatfield's investigations. It started in 1984 with the work of Van der Weijde (1984, 1985a,b). He did experiments on a 0·05 m diameter PVC pipeline containing a 24 m long U-shaped test section. An adjustable spring was used to vary the stiffness of the system. For less stiff systems he found considerable discrepancies between the experimental results and predictions obtained with conventional waterhammer theory. For this reason Delft Hydraulics decided to study FSI more thoroughly. A project was set up in co-operation with and sponsored by participants from industry, public authorities and consulting engineers' offices (FLUSTRIN 1986). First numerical results on a three-dimensional pipeline were presented in Tijsseling & Lavooij (1990). Kruisbrink (1990) attributed the observed damping of pressures in tests with relief valves to FSI effects, and verified his assertion through simulations with the FSI computer-code of Lavooij & Tijsseling (1989). Lavooij & Tijsseling (1991) applied two techniques to solve the basic equations in the time domain: the *MOC-FEM procedure* and the *MOC procedure.* The MOC-FEM procedure, which includes Poisson coupling, corresponds to the component-synthesis method of Wiggert & Hatfield (1983), the MOC procedure to the approach in Wiggert *et al.* (1985b). The two solution procedures were compared in the numerical studies (Heinsbroek *et al.* 1991; Heinsbroek & Tijsseling 1993). Laboratory tests on a scale larger than usual were conducted in order to validate the developed computer code (Kruisbrink & Heinsbroek 1992a,b; Heinsbroek & Kruisbrink 1993). The FSI test rig, Figure 7(a), consisted of a 77·5 m long, 0·11 m diameter, steel pipeline with six mitre bends, allowing for significant axial, flexural and torsional motion. Heinsbroek (1993) showed that, for this system, classical and uncoupled calculations render unreliable results. Pressure amplitudes and frequencies as a function of rigidity of the same system were studied numerically in Heinsbroek & Tijsseling (1994). In one example, FSI caused Joukowsky's value (1) to be exceeded by 100%, Figure 7(c).

Research at other institutes (continued)

Kojima *et al.* (1986) studied a straight pipe, using a four-equation model with Poisson coupling from fluid to pipe. Experimental data on an oil-filled pipe were provided.

Obradović (1990a,b,c) used the MOC method of Wiggert *et al.* (1987a) in solving a fourteen-equation model. In Obradović (1990b) an accident was simulated.

Elansary & Contractor (1990, 1993) applied extended waterhammer theory (four-equation model) to the problem of valve stroking. They described a procedure for the optimum closure of a valve in a given time interval to minimize reaction forces. Elansary *et al.* (1994) compared results of numerical simulations with Poisson coupling to measurements in a 32 m long, 0·10 m diameter, steel test section of a reservoir-pipe-valve system. Experimental data were used as input to the simulations.

The FSI computer code of Bettinali *et al.* (1991) is based on a solution technique similar to the MOC-FEM procedure of Lavooij & Tijsseling (1989), although the Poisson coupling is modelled in a different way. Some numerical results with respect to a simple single-pipe system subjected to seismic excitation were shown.
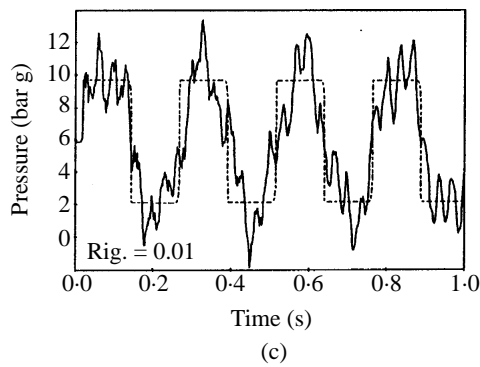
De Almeida & Koelle (1992, pp. 60–61) showed work of Vasconcellos (1991) on a
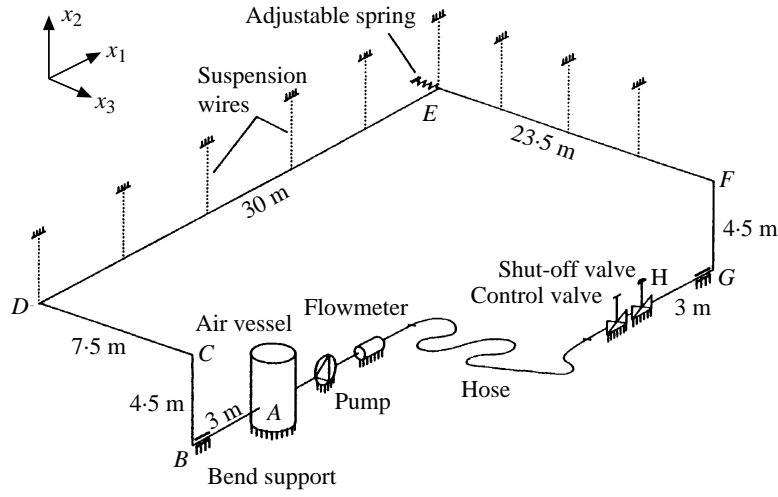
(a)
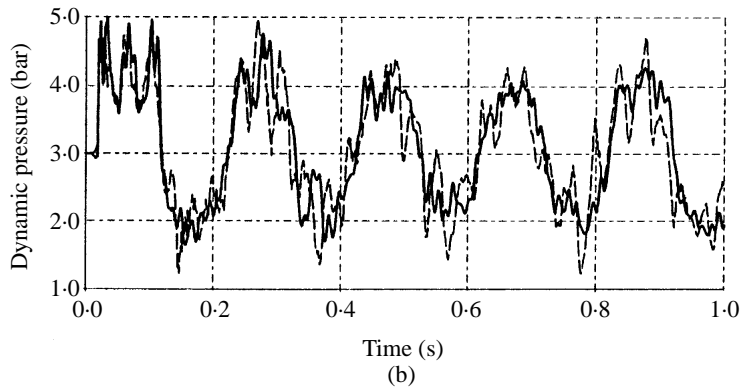
(b)

(c)

Figure 7. (a) Delft Hydraulics FSI test circuit (Kruisbrink & Heinsbroek 1992a). (b) Measured and computed dynamic pressure at the shut-off valve: ———, experiment; – – –, simulation; (Kruisbrink & Heinsbroek 1992a). (c) Computed pressure histories at the shut-off valve for the same system with slightly restrained bends: ———, extended waterhammer theory (with FSI); – – –, classical waterhammer theory (Heinsbroek & Tijsseling 1994).

reservoir-pipe-valve system, in which classical waterhammer predictions were compared with results calculated with Poisson and friction coupling.

Zhang *et al.* (1994) used the FEM for both structure and liquid (FEM procedure) to solve the four-equation model. Svingen (1994) used the FEM procedure in a frequency-domain analysis with Poisson and one-way junction coupling. Davidson & Smith's (1969) experimental results were used for a qualitative validation.

Commercial software, based on the literature described in this subsection and emphasizing the importance of the subject, is available from ESDU in London, U.K. (ESDU 1989, 1994), and Delft Hydraulics in Delft, The Netherlands (FLUSTRIN 1990).

3.2.6. *FSI analysis by coupling or extending existing software*

The literature dealt with in the preceding subsection is closely related to the present author's work. Fluid and structure are modelled one-dimensionally, and junction and Poisson coupling are taken into account. Some other literature is reviewed here. In contrast to the previous subsections, where most of the contributions came from universities, here work is discussed that comes from the practice of, in particular, nuclear power stations. The general approach is to use existing computer codes based on the finite-element method (FEM) to model the pipe system. Waterhammer loads are calculated by means of existing computer codes as well.

FSI is simulated by coupling the fluid and structural codes. At this point it is worthwhile to cite Belytschko *et al.* (1986, p. 249): "*Although the marriage of such fluid and structural analyses programs for the purpose of conducting fully coupled fluid-structure computations of the waterhammer response are possible in principle, the awkwardness and large size of such couples makes them quite unattractive. Furthermore, the amount of computer time which would be required to run such coupled analyses would be quite exorbitant, for in addition to the large number of computations, the system would have to cope with the problem of transferring in and out of fast core the structural and fluid analysis programs for each time step*".

Data transfer is one of the difficulties when coupling existing software. In many cases, human interference is necessary. In other cases, special software has to be developed (Ware & Williamson 1982). For *uncoupled calculations* (see Section 2.3) data transfer is one-way traffic, from fluid to structure. The fluid calculation precedes the structural calculation. Typical examples were given by Meder & Nguyen-Tuong (1984), Bühl (1987) and Hankinson & Van Duyne (1987). When FSI is taken into account, fluid and structural calculation must run simultaneously, with data transfer and possible iterations each time step. In the following, a selection of relevant time-domain literature is given.

Research in the U.S.A.

FSI experiments performed at Stanford Research Institute (SRI) under contract to Argonne National Laboratory (ANL) have been simulated by several investigators using different mathematical models and solution procedures. The experimental apparatus comprised one water-filled nickel test pipe of $1 \cdot 5$ m length, $0 \cdot 07$ m diameter
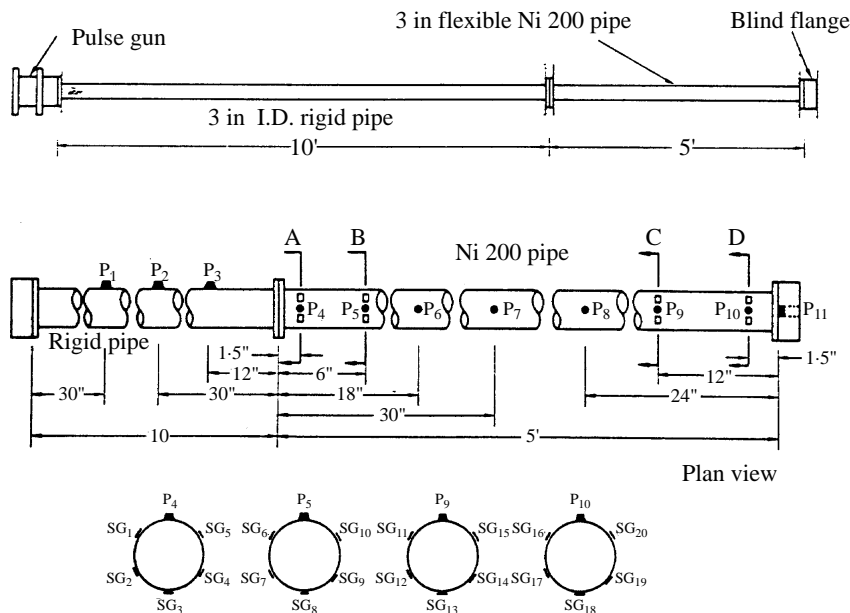
Figure 8. Schematic of the straight flexible pipe test configuration and location of instruments: (a) straight-pipe configuration; (b) instrumentation for straight-pipe text. P = pressure transducer, SG = strain gauge, (A-Moneim & Chang 1978).

and 0·0017 m wall thickness (Figure 8), or two of these pipes connected by a rigid elbow. The pipe ends and the elbow were fixed to the ground. At one of the closed ends, a pressure pulse of 3 ms duration and 0·2 ms rise time was generated by means of an explosion, representing a hypothetical core disruptive accident or a liquid sodium-water reaction in a nuclear plant. The pressure pulse reached values up to 16 MPa, causing local plastic wall deformation in the nickel pipe, which was extensively instrumented with pressure transducers and strain gauges.

A-Moneim & Chang (1978, 1979) simulated both the straight-pipe and one-elbow experiment, using a two-dimensional computer code allowing for fluid-structure coupling. The fluid equations were solved by a finite-difference method, the pipe equations by the FEM with axially symmetric thin-shell elements. Precursor effects, predicted for both fluid and pipe wall, were observed in the pipe wall only. Romander *et al.* (1980), Youngdahl *et al.* (1980), Wiedermann (1982) and Kulak (1982, 1985) simulated the straight-pipe experiment. For the pipe, Romander *et al.* (1980) used a two-dimensional model similar to that of A-Moneim & Chang, but solved the fluid equations by the FEM. Youngdahl *et al.* (1980) extended a conventional one-dimensional MOC waterhammer code with a simple ring-hoop deformation model for the pipe. Wiedermann (1982) employed a similar quasi-static interaction model. Numerical dispersion was presented as physical dispersion (in the opinion of the present author). Kulak (1982, 1985) utilized a FEM code with three-dimensional elements for both fluid and pipe. It is noted that all the quoted investigators used a measured pressure pulse as input to their calculations and that junction coupling was not involved. Their work focused on plastic wall deformation and/or Poisson coupling.

MOC-FEM procedure

Junction coupling, but not Poisson coupling, was taken into account by Giesecke (1981), Kellner & Schönfelder (1982) and Kellner *et al.* (1983). Giesecke applied the

standard MOC to the one-dimensional fluid equations and used beam elements to model the pipes within the FEM. He validated his results against the unreliable (see subsection 3.2.4) measurements of Swaffield (1968–1969). Kellner & Schönfelder (1982) coupled a fluid and a structural computer code and showed that neglecting FSI may lead to unrealistic predictions. The fluid code was based on the Lax-Wendroff method to allow for two-phase and wall-plasticity effects. In Kellner *et al.* (1983) convincing validation tests were presented for three pipe systems: a system with one elbow, a system with one branch, and a three-dimensional system with four elbows. The length of the latter system was 13·4 m; the pipe diameters were 0·10 m.

FEM procedure

Structural engineers determine the dynamic behaviour of pipe systems with the aid of general-purpose FEM computer codes. For them, it is attractive to solve the fluid equations with the same FEM code, especially when FSI has to be taken into account. In principle this is possible. For example, Rachford & Ramsey (1975), Watt *et al.* (1980) and Bach & Spangenberg (1990) applied the FEM to the classical waterhammer equations. Romander *et al.* (1980) and Kulak (1985), already mentioned, solved the fluid equations by means of the FEM as well. Their models were two- and three-dimensional, respectively, and junction coupling was neglected. Investigators applying the FEM to one-dimensional fluid models are considered next.

   Howlett (1971) modelled the liquid contained within a pipe system by means of beams without bending stiffness. Schwirian & Karabin (1981a,b), Schwirian (1982, 1984), Axisa & Gibert (1982), Everstine (1984, 1986) and Belytschko *et al.* (1986) followed the same approach and included elbows, tees and diameter changes in their work. The pipe system was modelled by beam elements; junction coupling was included, but Poisson coupling was not. Schwirian (1982, 1984) additionally presented a multi-dimensional fluid element with Poisson coupling incorporated. Axisa & Gibert (1982) gave a thorough mathematical treatment of their method. Everstine (1984, 1986) compared predictions of a one-dimensional fluid and pipe model, without Poisson coupling, with those of a three-dimensional model [like Kulak's (1982, 1985)] and with the experimental results of Davidson & Smith (1969). Belytschko *et al.* (1986) introduced silent boundaries in order to analyse small portions of complex pipe systems. Thomas (1991) gave a one-dimensional FEM treatise of fluid and structure in which the interaction was formulated in terms of local sources (Gibert *et al.* 1978). Just as Axisa & Gibert (1982) and Everstine (1981, 1984, 1986), Thomas derived a symmetric coefficient matrix by taking the displacement or velocity potential, rather than the pressure, as the fundamental fluid-unknown. It is interesting to note that within the context of a FEM formulation of waterhammer, Joukowsky's formula (1) is regarded as a dashpot equation.

Research in Germany

In Germany, an impressive research program with respect to the safety of nuclear power stations started in the mid-1970s. The program was carried out by the Karlsruhe Nuclear Research Centre (KfK—Kernforschungszentrum Karlsruhe) on behalf of the German government. The general objective of the program was the experimental validation of calculation tools and procedures. For this purpose, a full-scale test facility was available, the superheated steam reactor (HDR—Heißdampfreaktor) in Kahl near
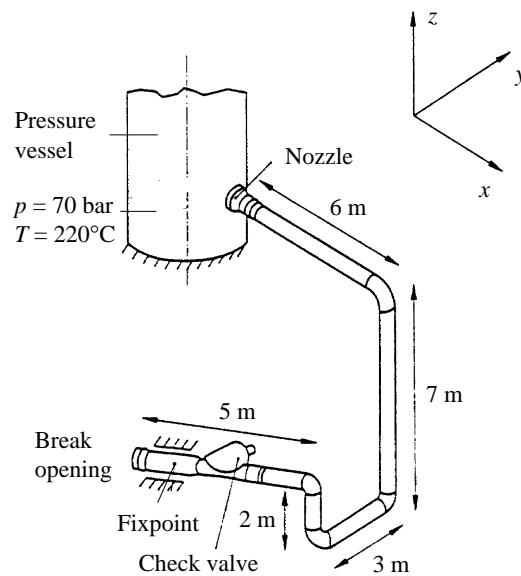
Figure 9. Configuration of the HDR test pipe (Müller 1987a).

Frankfurt, a former 100 MW nuclear plant. The experimental work was performed exclusively by KfK, whereas the computations were undertaken by several specialized institutions. Comparison and certification of software was part of the program. Pipe systems, empty and filled, straight and curved, branched and unbranched, were extensively investigated for static and dynamic, periodic and transient, elastic and plastic, as well as failure and crack behaviour. Results have been presented at all the SMiRT (Structural Mechanics in Reactor Technology) conferences since 1977.

   One test configuration, defined as a German Standard Problem, is of particular interest here. It comprises a three-dimensional pipe system of 23 m length, 0·40 m diameter and 0·02 m wall thickness, including four elbows (Figure 9). The system is filled with subcooled water of 7–9 MPa pressure and about 500 K temperature. In one of the tests, transients are generated by a check-valve closure due to a simulated pipe break. Some publications dealing with this specific test are considered now. Bietenbeck *et al.* (1985) simulated an earlier version of the aforementioned experiment. In an uncoupled calculation, measured pressure histories were used as input to a FEM code in which beam elements represented the pipe system. Müller (1986, 1987a) performed uncoupled calculations as well. Predicted pressure histories, obtained from a one-dimensional two-phase-flow computer code based on the MOC, were used as input to a FEM code similar to that of Bietenbeck *et al.* In Müller (1987b) FSI effects were taken into account. Results of coupled and uncoupled analyses were compared, but not validated against experimental data. The underlying calculation procedure was described by Grillenberger (1985), who demonstrated the significance of FSI in a simple reservoir-pipe-valve system. Firnhaber & Müller (1987) evaluated the pipe responses computed by 11 German engineering firms. The firms were provided with measured pressures, whilst the measured structural response was kept secret. The results of their uncoupled calculations showed differences which were mainly due to the structural code used, the way the measured pressures were incorporated, and the insight of the analyst. Kussmaul *et al.* (1989) performed an uncoupled calculation with measured data as input. Membrane-type pipe elements were used in a standard FEM

code. Plasticity was taken into account since, after a certain period of time, local yielding occurred in the experiment as a consequence of pressure peaks with observed magnitudes up to 31 MPa. It is noted that measured pressures contain the influences of FSI when present. By using these measured pressures in the simulations, FSI effects are tacitly introduced. Malcher & Steinhilber (1991) presented a review of the seismic tests conducted at the HDR test facility. It was reported that 25 institutions contributed to the project with analytical work.

3.2.7. *Fluid-structure interaction and cavitation*

Most FSI researchers ignore cavitation in their theoretical work and prevent cavitation in their experimental work. Research examining this extra complication is discussed in this subsection.

A-Moneim & Chang (1978, 1979) reported the occurrence of cavitation in the SRI experiment described in the previous subsection. They accounted for this phenomenon by taking a simple pressure cut-off level in their numerical model. Youngdahl *et al.* (1980) and Wiedermann (1982) applied the *concentrated* (or *discrete*) *cavity model* (Thibessard 1961; Provoost 1976; Kot & Youngdahl 1978a,b) in simulating the SRI experiment. FSI was taken into account only when the pipe was plastically deforming. Kot *et al.* (1980, 1981) analysed cavitation in combination with junction coupling. A thin spherical end cap interacted with the pressure waves in a liquid-filled pipe. The pressure waves were described by classical waterhammer theory in combination with the concentrated cavity model; the end cap was modelled as a thin shell within the FEM. Experimental validation was not given. Giesecke (1981) mentioned the concentrated cavity model but did not show any results. Axisa & Gibert (1982) and Schwirian (1982, 1984) employed the same model within the context of the FEM; gaps were allowed to form between the axial beam elements simulating the liquid. They compared numerical results obtained with and without cavitation. Van der Weijde (1985b) showed results of experiments with FSI and cavitation. All his calculations were without FSI.

In the HDR experiment described in subsection 3.2.6, subcooled water of 7–9 MPa pressure and 500 K temperature was used. When depressurizing the water, cavitation occurred, that is, a steam–water mixture formed. Müller (1987b) simulated this phenomenon with the aid of a two-phase-flow computer code including thermodynamic effects. He coupled the code to a structural FEM code, thereby including FSI, although it is not exactly clear to the present author which coupling mechanisms he took into account. Müller stated that FSI is unimportant as long as the two-phase conditions prevail, since the compressibility of the fluid is then vastly greater than the elasticity of the pipes. The present author agrees with this point of view in respect of Poisson coupling, but disagrees in respect of junction coupling. Furthermore, in two-phase flows, shock waves may develop which impose severe loads on the pipes. Müller did not compare his computed results with the available experimental data of Müller (1986, 1987a).

Tijsseling & Lavooij (1989) presented results of calculations with FSI and one column separation in a reservoir-pipe-valve system. The column separation occurred at the axially vibrating closed valve. In Tijsseling & Fan (1992) FSI and two column separations in a closed pipe were numerically simulated. Excellent agreement was found between simulation and experimental data obtained in Vardy & Fan's (1989) test rig. The same experimental data were used to validate the FSI four-equation model combined with the concentrated cavity model (Tijsseling & Fan 1991a,b). Fan &

Tijsseling (1992) predicted and observed *Poisson-coupling-induced cavitation,* that is cavitation caused by the radial expansion of the pipe wall. It was interesting that in all the laboratory tests, *tensile stresses* (Trevena 1987) were measured in the liquid (ordinary tap water) preceding the initiation of cavitation. Tijsseling *et al.* (1994, 1996) presented experimental and numerical results on a one-elbow pipe system, where the concentrated cavity model was incorporated in the FSI eight-equation model. Tijsseling (1993) gave a complete description of all experiments and simulations performed.

Bettinali *et al.* (1991) used the concentrated cavity model in their FSI computer-code. Results of one numerical test case were presented.

3.2.8. *Other mechanisms of fluid-structure interaction*

Bends and Bourdon coupling

In the majority of the work mentioned in the subsections 3.2.4 and 3.2.5, junction coupling at bends or elbows was modelled through the application of continuity and equilibrium principles to a control volume enclosing the entire junction (Figure 10). This simple model is valid if the length of the elbow is small compared to the lengths of the adjacent pipes. The angle between the pipes remains constant; elbow ovalization and the associated flexibility increase and stress intensification (Bantlin 1910; Von Kármán 1911; Hovgaard 1926) are ignored. However, these matters can be accounted for by flexibility and stress intensification factors (Vigness 1943; Clark & Reissner 1951; Dodge & Moore 1972). Bathe & Almeida (1980, 1982a,b) developed an accurate finite-element model of an elbow, which also includes the increased stiffness due to the internal pressure.

Bourdon coupling occurs in curved fluid-filled tubes of non-circular cross-section. Internal fluid pressure unbends the tube and externally imposed bending changes the fluid flow. The mechanism, used in Bourdon gauges to measure liquid pressures, is
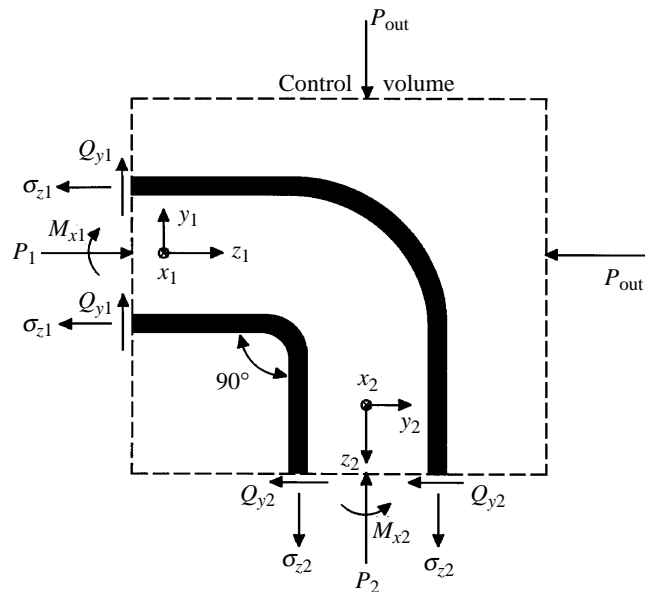

Figure 10. Definition sketch of an elbow junction (Tijsseling 1993).

probably best known from unrolling paper party tooters. Tentarelli (1990) was the first to investigate Bourdon coupling in a dynamic FSI analysis. His calculations in the frequency domain were confirmed by physical experiments in a J-shaped tube of slightly oval cross-section. The Bourdon effect results from cross-sectional area changes, which can also contribute to the liquid-pipe coupling in the flexure of a closed straight pipe, as observed by Budny (1988; Appendix A) and Fan (1989; p. 76).

Buckling and flutter

For the sake of completeness, mention should be made of another mechanism of fluid-structure interaction in liquid-filled pipes. This mechanism has little to do with fluid transients, but it concerns dynamic instabilities which lead to violent flexural vibrations of liquid-conveying pipes. The phenomenon occurs when the steady flow velocity reaches a certain limit, the critical velocity, at which the system loses stability by either buckling or flutter. "*In general, the critical flow velocity is very high and is of no great concern in practice*" (Jendrzejczyk & Chen 1984; p. 145). Fluid-structure interaction is induced by the fluid centrifugal force and the fluid Coriolis force. The system behaviour is highly dependent on the pipe support conditions. Some relevant literature on the subject is: Housner (1952), Gregory & Païdoussis (1966), Païdoussis & Issid (1974), Païdoussis & Laithier (1976) and Jendrzejczyk & Chen (1984, 1985). Historical reviews were given by Païdoussis & Issid (1974) and Païdoussis & Li (1993). The latter review emphasizes the interesting nonlinear dynamics phenomena involved.

4. REVIEW AND CONCLUSION

The history of research on transient phenomena in liquid-filled pipe systems has been surveyed. Important developments were the classical waterhammer investigations of Joukowsky and Allievi around 1900 and the recognition of the effects of fluid-structure interaction (FSI) in the 1960s.

The basic equations underlying present-day FSI analyses were derived in the nineteenth century. In the 1950s the first analytical solutions to these, sometimes extended, equations appeared. Wave propagation modes including liquid-pipe coupling mechanisms were calculated. Skalak (1956) predicted the existence of *precursor waves,* which were confirmed experimentally by Thorley (1969). The significant influence of *unbalanced pressure forces,* acting on unrestrained dead-ends, elbows and tees, on the dynamic behaviour of liquid-filled pipe systems was clearly demonstrated in the 1960s.

The first digital computer codes for solving waterhammer problems were developed in the 1960s. In the 1970s, the basic FSI equations could be solved numerically and the nuclear power industry gave FSI research a big push forward. In the 1980s, the trend was to couple existing liquid-flow and pipe-stress software in an iterative manner. In the 1990s, the first special-purpose commercial codes directed to FSI have been released.

A one-dimensional *fourteen-equation model* (see subsection 3.2.5) describing the axial (longitudinal), lateral (flexural) and torsional motion of thin-walled liquid-filled pipe systems is adequate for most practical purposes. It can be enhanced: (i) with flexibility and stress-intensification factors to allow for elbow ovalization effects and (ii) with the concentrated (discrete) cavity model to allow for column separation and vaporous cavitation. The model includes all relevant FSI mechanisms and is valid for long-wavelength (low-frequency) acoustical phenomena. Radial inertia forces are assumed negligible in the long-wavelength approximation.

The method of characteristics (MOC) and the finite-element method (FEM), or a combination of both, are the most common numerical methods for solving the one-dimensional basic equations. Two- and three-dimensional models are sometimes used in applications where high accuracy is required or to provide reference solutions for the verification of one-dimensional models.

The present selection of literature emphasises *inter*action, that is two-way coupling, in the time domain. Although the review is comprehensive, the author does not claim that it is complete. For example, important work has been done in the frequency domain, placing emphasis on sensitivity to noise, vibration and seismic excitation.

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Abbreviations

ASCE   American Society of Civil Engineers
ASME   American Society of Mechanical Engineers

BHRA   British Hydromechanics Research Association
BNES   British Nuclear Energy Society
FED   Fluids Engineering Division
IAHR   International Association for Hydraulic Research
IMechE   Institution of Mechanical Engineers
PVP   Pressure Vessels and Piping
3R   Rohre Rohrleitungsbau Rohrleitungstransport
SMiRT   Structural Mechanics in Reactor Technology

APPENDIX: NOMENCLATURE

A	cross-sectional area
c	classical wave propagation speed
$\tilde{c}$	Poisson-coupled wave propagation speed
c_1	wave propagation speed in unconfined fluid
c_2	wave propagation speed in highly elastic tube
D	inner diameter of pipe
E	Young's modulus of elasticity
e	pipe wall thickness
G	shear modulus
I	second moment of cross-sectional area
J	polar second moment of cross-sectional area
K	liquid bulk modulus
M	bending moment, torsional moment
P	pressure (cross-sectional average)
Q	lateral shear force
R	inner radius of pipe
t	time
$\dot{u}$	pipe velocity
V	fluid velocity (cross-sectional average)
z	distance along pipe
$\dot{\theta}$	rotational velocity of pipe
κ^2	shear coefficient
ν	Poisson ratio
ρ	mass density
σ	normal stress
ψ	correction factor in formula for pressure wave speed

Subscripts

f	fluid
out	outer
r	radial direction
t	tube, pipe
x	lateral direction
y	lateral direction
z	axial direction
ϕ	circumferential direction, hoop