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POISSON-COUPLING BEAT IN EXTENDED WATERHAMMER THEORY

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ABSTRACT

The axial/radial vibrations of liquid-filled pipes, excited by long-wave waterhammer, are adequately described by the so-called *four-equation model*. The model is based on *FSI Poisson coupling*, which can produce an interesting and new phenomenon: the Poisson-coupling beat. The present paper introduces this phenomenon, shows its theoretical evidence and discusses its practical consequences.

1. Introduction

Skalak's (1955/1956a, 1956b) *extended theory of waterhammer* describes axisymmetric wave propagation in straight liquid-filled pipes. Skalak presented two mathematical models: a complex one and a simple one. The simple model is popular with investigators of fluid-structure interaction (*FSI*) in liquid-filled pipes (Tijsseling 1996). It permits the non-dispersive, but coupled, propagation of pressure waves in the liquid and axial stress waves in the pipe wall. The FSI coupling mechanism is often referred to as *Poisson coupling*, because it is induced by axial/radial pipe contractions which are proportional to Poisson's ratio ν.

Skalak used non-equilibrium initial conditions to initiate wave propagation in (semi)infinite pipes. He did not consider reflections from pipe ends. However, when pipe end conditions are included in the analysis, an interesting phenomenon may occur: the *Poisson-coupling beat* (Wiggert *et al.* 1986).

The present paper highlights this phenomenon. It is intended to invalidate the common belief that Poisson coupling is relatively unimportant, but at the same time it questions the validity of Skalak's simple model.

2. FSI four-equation model

Skalak's (1955/1956a, 1956b) simple model can be represented by the four equations

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f}\frac{\partial P}{\partial z} = 0 \tag{1}$$

$$\frac{\partial V}{\partial z} + \left(\frac{1}{K} + \frac{2R}{Ee}\right)\frac{\partial P}{\partial t} - \frac{2\nu}{E}\frac{\partial \sigma_z}{\partial t} = 0 \tag{2}$$

$$\frac{\partial \dot{u}_z}{\partial t} - \frac{1}{\rho_s}\frac{\partial \sigma_z}{\partial z} = 0 \tag{3}$$

$$\frac{\partial \dot{u}_z}{\partial z} - \frac{1}{E}\frac{\partial \sigma_z}{\partial t} + \frac{\nu R}{Ee}\frac{\partial P}{\partial t} = 0 \tag{4}$$

governing the unknowns: fluid pressure, P; fluid velocity, V; axial pipe stress, σ_z; and axial pipe velocity, $\dot{u}_z$. Notation: E = Young's modulus, e = wall thickness, K = bulk modulus, R = inner pipe radius, t = time, z = distance along pipe, ρ = mass density; the subscripts f and s refer to fluid and structure, respectively. The model is valid for the low-frequency acoustic behaviour of straight, thin-walled, linearly elastic, liquid-filled, prismatic pipes of circular cross-section.

The wave speeds, c, are found from the *characteristic equation*

$$\left\{\frac{\rho_f}{K} + (1-\nu^2)\frac{2R\rho_f}{eE}\right\}c^4 - \left\{1 + \left(\frac{2R}{e} + \frac{E}{K}\right)\frac{\rho_f}{\rho_s}\right\}c^2 + \frac{E}{\rho_s} = 0 \tag{5}$$

belonging to the four-equation model (1)-(4).

3. Precursor waves

Precursor waves are stress-wave-induced disturbances in the liquid which travel ahead of the classical waterhammer waves. Skalak (1955/1956a, 1956b) predicted the existence of precursor waves, which were clearly measured by Thorley (1969), Vardy and Fan (1986) and Kojima and Shinada (1988).

Although, in water-filled steel pipes, precursor waves are generally very small and fast when compared to classical waterhammer waves, their cumulative effect can be significant. In fact, Poisson-coupling beat is caused by precursor waves.

4. Poisson-coupling beat

Wiggert *et al.* (1986) predicted Poisson-coupling beat in a reservoir - straight-pipe - valve system with end conditions that were *open-closed* for the liquid and *fixed-fixed* for the pipe. The system was excited by *instantaneous* valve closure and the ratio of stress wave speed, c_s , to pressure wave speed, c_f , was an integer number in their calculations. Tijsseling & Lavooij (1989) predicted the same phenomenon with c_s/c_f being a rational number. Schwarz (1978) was not able to detect the Poisson-coupling beat, because of a too slow valve closure in his calculations. Heinsbroek & Tijsseling (1994) computed the phenomenon in a 3-D pipeline with stiffly supported bends. Poisson-coupling beat resulting from non-instantaneous valve closure is shown in Section 5 of this paper.

5. Theoretical evidence

The reservoir-pipe-valve system (Fig. 1) from (Tijsseling & Lavooij 1989) was defined as an FSI benchmark theoretical problem and it is used herein to demonstrate Poisson-coupling beat. The liquid-filled pipe is unrestrained throughout its length, but fixed at its ends. The L = 20 m long system is characterised by wave speeds c_f = 1024.6 m/s and c_s = 5280.5 m/s in the liquid and pipe wall, respectively, with corresponding timescales $T_f = 2L/c_f$ = 39.0 ms and $T_s = 2L/c_s$ = 7.6 ms. The valve stops within T_c = 50 ms an initial flow of V_0 = 1 m/s.

The present result has been obtained with MOC calculations (Δt = 0.29 ms, Δz = 20 m, c_s/c_f = 67/13 and results stored every 13 time steps), but could equally well be obtained from a MOC-FEM analysis (Heinsbroek *et al.* 1991).

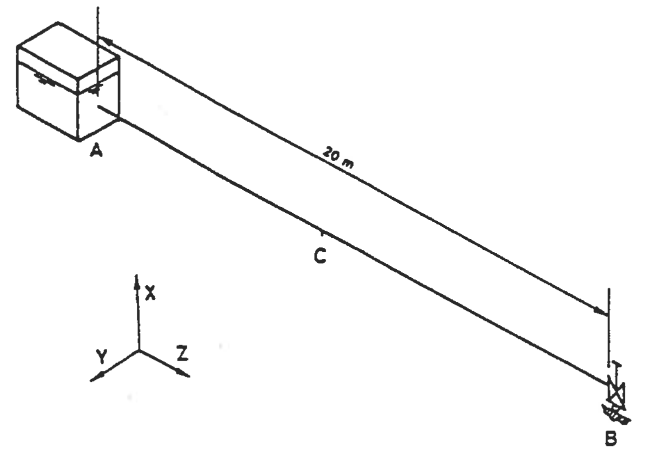
The piezometric-head history at the valve (Fig. 2) exhibits the beat phenomenon, the focal point of this paper. The beat period is about 1.5 s. The author has not yet been able to formally derive an analytical expression predicting this value. However, assuming that the beat period depends on the waterhammer period $T_w = 4L/c_f$ and on the ratio of wave speeds c_s/c_f, the following is found: 4 $T_w\, c_s/c_f$ = 1.6 s. This formula has been confirmed by the simulation of the *closed-closed* liquid, fixed-fixed pipe, system of Kuiken (1988) for which the beat period is 4 $T_f\, c_s/c_f$.

The first waterhammer cycle in the history shows that the valve closure generates the full Joukowsky head of $V_0\, c_f / g$ = 104.4 m. This is correct, because the effective rise time T_{cef} = 0.3 T_c = 15 ms is smaller than the wave return time T_f. The subsequent cycles have amplitudes of increasing magnitude, thereby violating the classical waterhammer assumption that Joukowsky's value is the maximum head achievable.

It is emphasised that Fig. 2 shows an exact solution of the four-equation model (1)-(4) if c_s/c_f = 67/13. No numerical approximations have been made in integrating the equations and the quadratic head/velocity relation describing the valve-closure has been solved exactly.

6. Conclusion and discussion

It has been shown, theoretically, that in a single straight pipe with open-closed and fixed-fixed boundaries for liquid and pipe respectively, FSI Poisson coupling strongly influences the impact-induced system vibration. The precursor waves in Skalak's (1955/1956a, 1956b) model are responsible for a



Length	=	20 m
Outer diameter	=	813 mm
Wall thickness	=	8 mm
Young's modulus	=	$21*10^{10}$ N/m^2
Density of pipe	=	7900 kg/m^3
Poisson's ratio	=	0.3
Density of fluid	=	1000 kg/m^3
Bulk modulus	=	$21*10^{8}$ N/m^2
Fluid velocity	=	1 m/s
Pressure behind valve	=	0 Pa
Gravitational acceleration	=	9.813 m/s^2

Figure 1 DELFT HYDRAULICS benchmark problem A

cumulative effect referred to as Poisson-coupling beat.

Precursor waves have been observed in laboratory experiments, but as far as the author knows, Poisson-coupling beat has not. Two obvious reasons are (i) pipe supports are never entirely stiff or entirely inert when loaded by impacts such as in the example presented (Heinsbroek and Tijsseling 1994) and (ii) damping occurs in practical systems at pipe supports, and in the liquid most likely by unsteady friction (Vardy and Brown 1996; Bughazem and Anderson 1996; Silva-Araya and Chaudhry 1997). Also, Skalak's simple model is not valid in the vicinity of pipe supports and near to steep wave fronts (Bürmann 1980, 1983; Adachi *et al.* 1991).

The four-equation model has been found to be highly accurate in previous work (Vardy and Fan 1989) and therefore the author believes that in a well set-up and controlled laboratory experiment the initial (transient) phase of Poisson-coupling beat, where damping does not play a role, can be generated.

In the application of FSI analyses to practical pipe systems, it is convenient to consider pipe supports as either entirely stiff (e.g. at pumps and reservoirs) or entirely flexible (e.g. at racks and clamps), because it is difficult and expensive to assess the dynamic characteristics of supports. If -- and this is quite common -- pipe supports are modelled as entirely stiff, instantaneous valve closure is chosen as the system excitation and the analysis allows for precursor waves, one should be aware that unrealistic pressure spikes may be predicted (Heinsbroek and Tijsseling 1994). In that sense, Poisson-coupling beat can be an annoyance for the analysing engineer.

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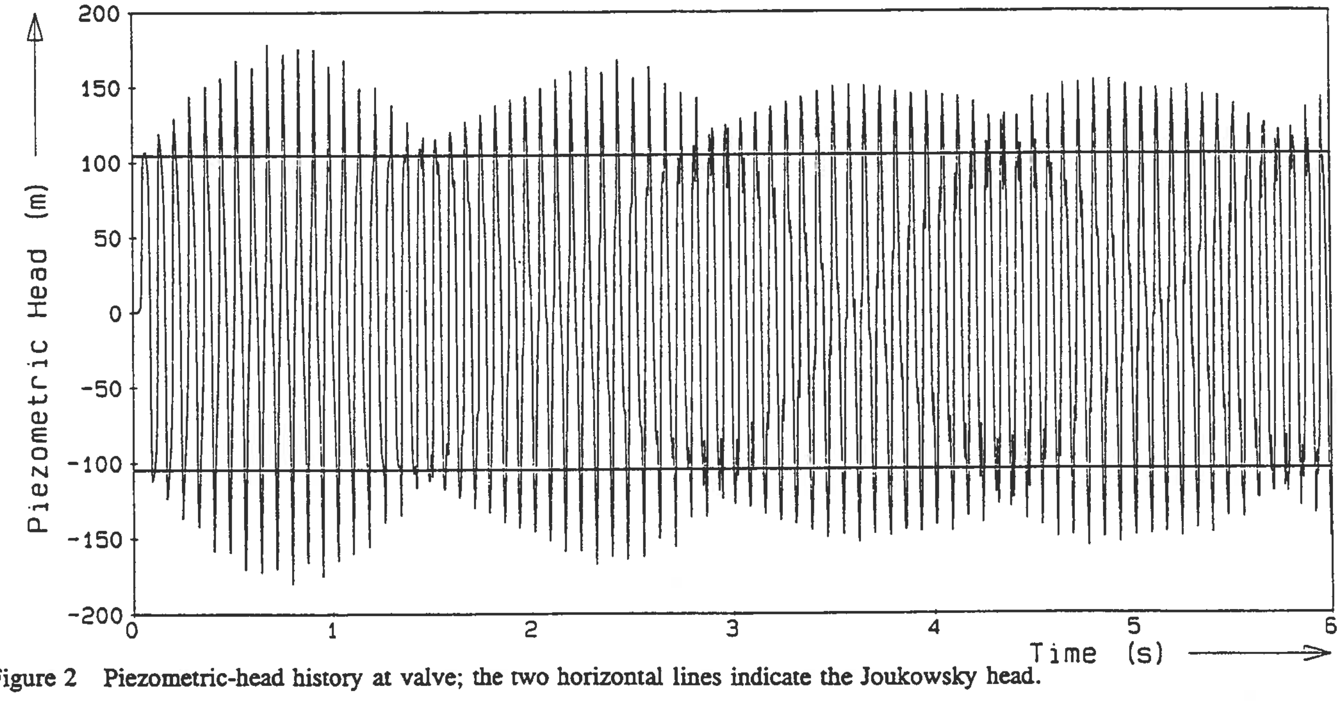

Figure 2 Piezometric-head history at valve; the two horizontal lines indicate the Joukowsky head.

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