2IF55 Semantics and computational models

Recursion—transition systems and higher-order definitions

Technische Universiteit Eindhoven

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Outline

J.W. de Bakker & E.P. de Vink, *Control Flow Semantics*
The MIT Press 1996

- Chapter 1: Recursion and Iterations
  Section 1.1: Recursion
- Chapter 2: Nondeterminacy
- Chapter 4: Uniform Parallelism
- Chapter 11: Branching Domains at Work

§1.1.3 Denotational semantics

denotational principles

- *program construct denote values*
- *the meaning of a whole*
  is composed from the meaning of constituents
- *infinite computation is dealt with by fixed points*

an operator for sequential composition

denotational domain $\mathbb{P}_D = \mathsf{Act}^\infty \setminus \{\epsilon\}$

$; : \mathbb{P}_D \times \mathbb{P}_D \xrightarrow{1} \mathbb{P}_D$ unique non-expansive function such that

$$
\begin{aligned}
;(a, p) &= a \cdot p \\
;(a \cdot p', p) &= a \cdot (\,;(p', p)\,)
\end{aligned}
$$

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put $\mathsf{Op} = \mathbb{P}_D \times \mathbb{P}_D \xrightarrow{1} \mathbb{P}_D$ and $\Omega_; : \mathsf{Op} \to \mathsf{Op}$

$$
\begin{aligned}
\Omega_;(\varphi)(a, p) &= a \cdot p \\
\Omega_;(\varphi)(a \cdot p', p) &= a \cdot (\,\varphi(p', p)\,)
\end{aligned}
$$

well-definedness of ;

lemma 1.37 $\Omega_;$ is $\frac{1}{2}$-contractive

lemma 1.38 $(p_1 ; p_2); p_3 = p_1 ; (p_2 ; p_3)$

lemma 1.41 $d(p_1 ; p_2, \bar{p}_1 ; \bar{p}_2) \leq \max\{\, d(p_1, \bar{p}_1),\ \frac{1}{2} d(p_2, \bar{p}_2)\,\}$

denotational mapping

put $\mathsf{Sem}_D = \mathcal{L}_{rec} \to \mathbb{P}_D$ and $\Psi \colon \mathsf{Sem}_D \to \mathsf{Sem}_D$

$$
\begin{aligned}
\Psi(S)(D|a) &= a \\
\Psi(S)(D|x) &= \Psi(S)(D|\, D(x)\,) \\
\Psi(S)(D|s_1; s_2) &= \Psi(S)(D|s_1); S(D|s_2)
\end{aligned}
$$

lemma 1.42 Ψ is $\frac{1}{2}$-contractive

denotational semantics $\mathcal{D} = \mathsf{fix}(\Psi)$

lemma 1.43

- $\mathcal{D}(D|(s_1; s_2); s_3) = \mathcal{D}(D|s_1; (s_2; s_3))$
- $\mathcal{D}(D|x) = a^\omega$ if $D(x) = a; x$