Assignment 1

a)
$$\frac{}{a.x \xrightarrow{a} x} \qquad \frac{x \xrightarrow{a} x'}{x + y \xrightarrow{a} x'} \qquad \frac{y \xrightarrow{a} y'}{x + y \xrightarrow{a} y'} \qquad \frac{x_N \xrightarrow{a} x \quad N = x_N}{N \xrightarrow{a} x}$$

$$\frac{}{\mathbf{1}\downarrow} \qquad \frac{x\downarrow}{x + y\downarrow} \qquad \frac{y\downarrow}{x + y\downarrow} \qquad \frac{x_N\downarrow \quad N = x_N}{N\downarrow}$$

(2 pts. per rule, for termination of $\mathbf{1}$ only 1 pts.)

b)
$$\frac{\dfrac{}{a.b.S \xrightarrow{a} b.S}}{\dfrac{a.b.S + T + \mathbf{1} \xrightarrow{a} b.S}{S \xrightarrow{a} b.S}} \qquad \frac{\dfrac{\dfrac{\dfrac{}{c.U \xrightarrow{c} U}}{c.U + U + \mathbf{0} \xrightarrow{c} U}}{T \xrightarrow{c} U}}{\dfrac{a.b.S + T \xrightarrow{c} U}{a.b.S + T + \mathbf{1} \xrightarrow{c} U}}{S \xrightarrow{c} U} \qquad \frac{\dfrac{}{\mathbf{1}\downarrow}}{\dfrac{(a.b.S + T + \mathbf{1})\downarrow}{S\downarrow}}$$

(6+6+8 pts., 6 deduction if other given)

Assignment 2

a) The relation $s \xrightarrow{w}\!\!\!\!\rightarrow t$ on $\mathcal{S} \times \mathcal{A}^* \times \mathcal{S}$ is inductively defined by: (i) $s \xrightarrow{\varepsilon}\!\!\!\!\rightarrow s$; (ii) if $s \xrightarrow{a} t$ and $t \xrightarrow{w}\!\!\!\!\rightarrow u$ then $s \xrightarrow{aw}\!\!\!\!\rightarrow u$; (iii) if $s \xrightarrow{\tau} t$ and $t \xrightarrow{w}\!\!\!\!\rightarrow u$ then $s \xrightarrow{w}\!\!\!\!\rightarrow u$. (2+1+2 pts.) The language $\mathcal{L}(\mathcal{M})$ is given by $\{\, w \in \mathcal{A}^* \mid \uparrow \xrightarrow{w}\!\!\!\!\rightarrow t \wedge t \in \downarrow \,\}$. (5 pts.)

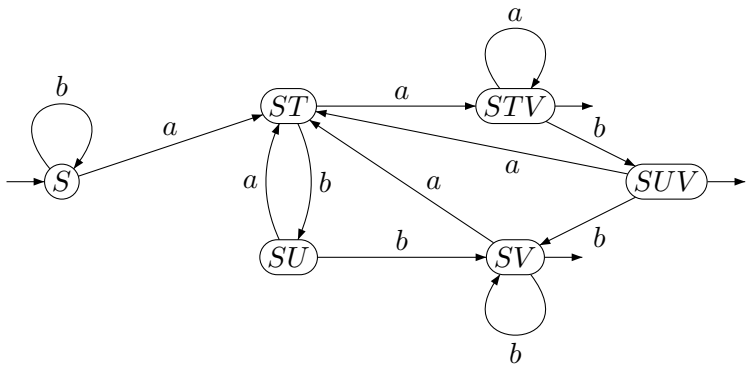
b) There exists no branching bisimulation connecting the two automata. (5 pts.) .
The state U is the problematic state. (5 pts.)
The initial states A and S need to be related by the branching bisimulation. The transition $S \xrightarrow{\tau} U$ is not inert as it looses the option of evolving into a menu of a only. Now, U cannot relate to A, since A can evolve into a menu of a-only. U cannot relate to B, for the same reason. U cannot relate to C, D and E since their menus are different from $\{a, b\}$. (10 pts.)

Assignment 3

a) The language accepted by the automaton is $\{\, waav,\ wabbv \mid w \in \{a,b\}^*,\ v \in \{b\}^* \,\}$. (10 pts., 5 pts. deduction if other parts given)

b)
$$\begin{array}{rcl} S &=& a.S + b.S + a.T \\ T &=& a.V + b.U \\ U &=& b.V \\ V &=& b.V + \mathbf{1} \end{array} \qquad \begin{array}{rcl} S &=& a.ST + b.S \\ T &=& a.V + b.U \\ U &=& b.V \\ V &=& b.V + \mathbf{1} \\ ST &=& a.STV + b.SU \end{array} \qquad \begin{array}{rcl} S &=& a.ST + b.S \\ T &=& a.V + b.U \\ U &=& b.V \\ V &=& b.V + \mathbf{1} \\ ST &=& a.STV + b.SU \\ STV &=& a.STV + b.SUV + \mathbf{1} \\ SU &=& a.ST + b.SV \end{array}$$

$$
\begin{aligned}
S &= a.ST + b.S \\
T &= a.V + b.U \\
U &= b.V \\
V &= b.V + \mathbf{1} \\
ST &= a.STV + b.SU \\
STV &= a.STV + b.SUV + \mathbf{1} \\
SU &= a.ST + b.SV \\
SUV &= a.ST + b.SV + \mathbf{1}
\end{aligned}
\qquad
\begin{aligned}
S &= a.ST + b.S \\
T &= a.V + b.U \\
U &= b.V \\
V &= b.V + \mathbf{1} \\
ST &= a.STV + b.SU \\
STV &= a.STV + b.SUV + \mathbf{1} \\
SU &= a.ST + b.SV \\
SUV &= a.ST + b.SV + \mathbf{1} \\
SV &= a.ST + b.SV + \mathbf{1}
\end{aligned}
\qquad
\begin{aligned}
S &= a.ST + b.S \\
T &= a.V + b.U \\
U &= a.X + b.V \\
V &= a.X + b.V + \mathbf{1} \\
ST &= a.STV + b.SU \\
STV &= a.STV + b.SUV + \mathbf{1} \\
SU &= a.ST + b.SV \\
SUV &= a.ST + b.SV + \mathbf{1} \\
SV &= a.ST + b.SV + \mathbf{1} \\
X &= a.X + b.X
\end{aligned}
$$



(5 pts. for linear specification, 5 pts. for expansion with combined variables, 5 pts. for completion with trap variable X or remark that the automaton is complete already in case it is, 5 pts. for final automaton (unreachable states allowed)