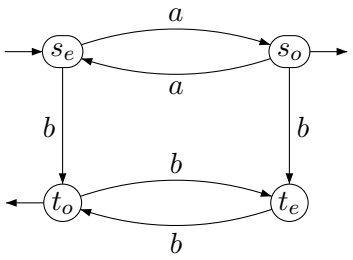
Assignment 1

a)



b) ($L \subseteq \mathcal{L}(M)$) Pick $a^n b^m \in L$. We distinguish three cases. (i) Suppose n is even, m is odd. We have $s_e \xrightarrow{a^n} s_e$, $s_e \xrightarrow{b} t_o$, $t_o \xrightarrow{b^{m-1}} t_o$ and $t_o\!\downarrow$. So, in this case, $a^n b^m \in \mathcal{L}(M)$. (ii) Suppose n is odd, $m = 0$. We have $s_e \xrightarrow{a^n} s_o$ and $s_o\!\downarrow$. So, in this case, $a^n b^m \in \mathcal{L}(M)$. (iii) Suppose n is odd, $m > 0$ is even. We have $s_e \xrightarrow{a^n} s_o$, $s_o \xrightarrow{b} t_e$, $t_e \xrightarrow{b^{m-1}} t_o$ and $t_o\!\downarrow$. So, also in this case, $a^n b^m \in \mathcal{L}(M)$.
($\mathcal{L}(M) \subseteq L$) Suppose $w \in \mathcal{L}(M)$, i.e. $s_e \xrightarrow{w} s_o$ or $s_e \xrightarrow{w} t_o$. In the first case we have $w = a^n$ for some odd n. Hence $w \in L$. In the second case we either have $s_e \xrightarrow{a^n} s_e \xrightarrow{b} t_o \xrightarrow{b^\ell} t_o$ with n even and ℓ even and $w = a^n b b^\ell$, or $s_e \xrightarrow{a^n} s_o \xrightarrow{b} t_e \xrightarrow{b^\ell} t_o$ with n odd and ℓ odd and $w = a^n b b^\ell$. In both cases we have $n + 1 + \ell$ is odd. Hence, $w \in L$.

Assignment 2

a) Two terms $x, y \in IA_\tau(\mathcal{N})$ are bisimilar iff their automata $\mathcal{M}(x)$ and $\mathcal{M}(y)$ are bisimilar.

b) Consider $(a.\mathbf{1} \cdot a.\mathbf{1})^*$ and $(a.\mathbf{1})^* \cdot (a.\mathbf{1})^*$. It holds that $(a.\mathbf{1})^* \cdot (a.\mathbf{1})^* \xrightarrow{a} (\mathbf{1} \cdot (a.\mathbf{1})^*) \cdot (a.\mathbf{1})^*$ and $(\mathbf{1} \cdot (a.\mathbf{1})^*) \cdot (a.\mathbf{1})^*\!\downarrow$. But $(a.\mathbf{1} \cdot a.\mathbf{1})^* \xrightarrow{a} (\mathbf{1} \cdot a.\mathbf{1}) \cdot (a.\mathbf{1} \cdot a.\mathbf{1})^*$ is the only a-transition for $(a.\mathbf{1} \cdot a.\mathbf{1})^*$ but we do not have $(\mathbf{1} \cdot a.\mathbf{1}) \cdot (a.\mathbf{1} \cdot a.\mathbf{1})^*\!\downarrow$. Thus $a \in \mathcal{L}((a.\mathbf{1})^* \cdot (a.\mathbf{1})^*)$ but $a \notin \mathcal{L}((a.\mathbf{1} \cdot a.\mathbf{1})^*)$. Hence, these two processes are not bisimilar and $(x \cdot x)^* \leftrightarrows x^* \cdot x^*$ does not hold in general.

Assignment 3

a) See Theorem 4.57, page 129.

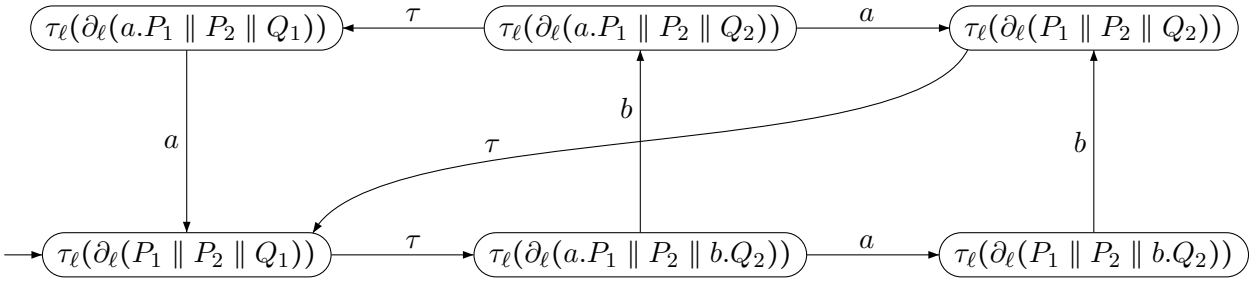
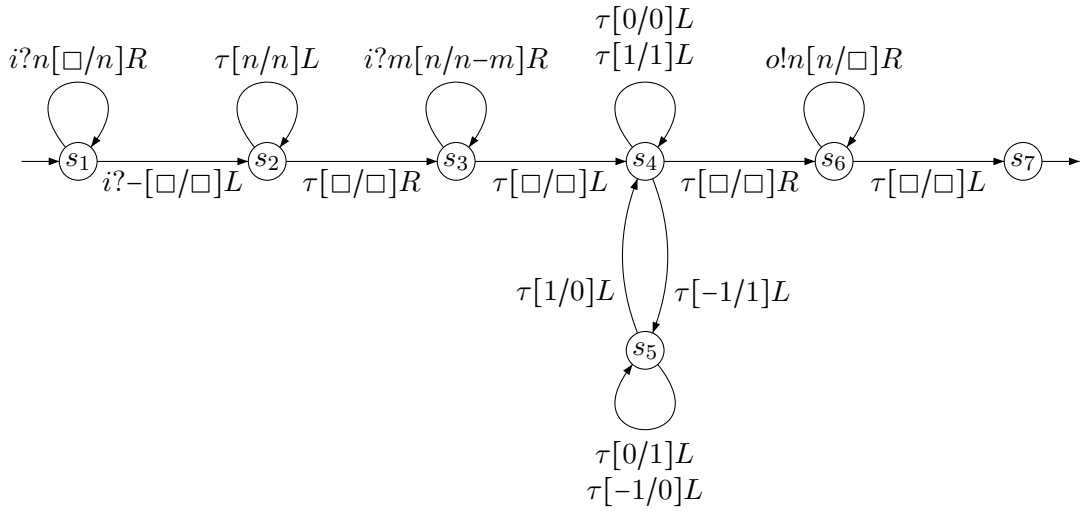
b) See Example 4.58, page 130.

Assignment 4

a)

$$\dfrac{\dfrac{\dfrac{\ell!1.a.P_1 \xrightarrow{\ell!1} a.P_1 \qquad P_1 = \ell!1.a.P_1}{P_1 \xrightarrow{\ell!1} a.P_1}}{P_1 \parallel P_2 \xrightarrow{\ell!1} a.P_1 \parallel P_2} \qquad \dfrac{\ell?1.b.Q_2 \xrightarrow{\ell?1} b.Q_2 \qquad Q_1 = \ell?1.b.Q_2}{Q_1 \xrightarrow{\ell?1} b.Q_2}}{\dfrac{P_1 \parallel P_2 \parallel Q_1 \xrightarrow{\ell\,\natural\,1} a.P_1 \parallel P_2 \parallel b.Q_2}{\dfrac{\partial_\ell(P_1 \parallel P_2 \parallel Q_1) \xrightarrow{\ell\,\natural\,1} \partial_\ell(a.P_1 \parallel P_2 \parallel b.Q_2)}{\tau_\ell(\,\partial_\ell(P_1 \parallel P_2 \parallel Q_1)\,) \xrightarrow{\tau} \tau_\ell(\partial_\ell(a.P_1 \parallel P_2 \parallel b.Q_2))}}}$$

b)



Assignment 5 Compare Example 5.12, page 143.