Assignment 1

a) Formulate the Pumping Lemma for regular languages. (10 pts.)

b) Prove that the language $L = \{ w \in \{a,b\}^* \mid \#_a(w) = \#_b(w) \}$ is *not* regular. (10 pts.)

Assignment 2

a) Give the operational rules of the iteration algebra $IA_\tau(\mathcal{N})$ for the sequential composition and the iteration construct. (Five in total.) (10 pts.)

b) Prove that $x^* \cdot (x + \mathbf{1}) \leftrightarrow x^*$, for all processes $x \in IA_\tau(\mathcal{N})$, by providing a bisimulation relation and verifying that it is a bisimulation relation indeed, or disprove that $x^* \cdot (x + \mathbf{1}) \leftrightarrow x^*$ by providing a counter example and arguing why the respective processes cannot be bisimilar. (10 pts.)

Assignment 3

a) Draw a push-down automaton M (with no more than 3 states) that accepts the language $L = \{ a^n b a^n \mid n \geqslant 0 \}$. (10 pts.)

b) Prove that $L = \mathcal{L}(M)$, either by verifying two set inclusions or by providing path invariants. (10 pts.)

Assignment 4 Consider the following processes of the process algebra $CA_\tau(\{P_1, P_2, Q_1, Q_2\})$, that includes parallel composition, the encapsultation operator and the abstraction operator:

$$\begin{aligned} P_1 &= \ell!1.a_1.P_1 \\ P_2 &= \ell!2.a_2.P_2 \\ Q_1 &= \ell?1.Q_2 \\ Q_2 &= \ell?2.Q_1 \end{aligned}$$

a) Give a derivation for the first transition of $\tau_\ell(\,\partial_\ell(P_1 \parallel P_2 \parallel Q_1)\,)$. (10 pts.)

b) Draw the transition system of $\tau_\ell(\,\partial_\ell(P_1 \parallel P_2 \parallel Q_1)\,)$ (10 pts.)

Assignment 5 Provide a Turing machine M with no more than six states that is able to compare binary numbers of equal length, i.e. give a Turing machine M that returns on input of the string $w > v$, with $w, v \in \{0,1\}^*$ such that $|w| = |v| > 0$, the answer *yes* if the binary number represented by w is larger than the binary number represented by v, and returns the answer *no* otherwise. (20 pts.)

The end marking is the total of accredited points divided by 10.