Special Sets

Set			Element	
Name	**Notation**	**Definition**	Name	**Notation**
alphabet	Σ	Enumeration ($\neq \emptyset$)	symbol letter	$a, b, \ldots$ arbitrary symbols
n-symbol strings over an alphabet	$\Sigma^n,\ n \geq 0$	finite product (see below)	string word	$a_1 a_2 \ldots a_n, \quad n \geq 0$ $a_1 a_2 \ldots a_0 = \varepsilon$
all finite strings over an alphabet	Σ^*	1. set union 2. Inductive	string word	ε, empty word $w, v, u, \ldots$ arbitrary words
language	L	subset of Σ^*	word	

Definition 1

$\Sigma^n = \{ a_1 a_2 \ldots a_n \mid \forall i, 1 \leq i \leq n : a_i \in \Sigma \}$

$\Sigma^* = \bigcup_{n=0}^{\infty} \Sigma^n$

Definition 2

1. The empty word $\varepsilon \in \Sigma^*$
2. If $a \in \Sigma,\ w \in \Sigma^*$, then $aw \in \Sigma^*$

Relations on Σ^*

Name	Notation	Definition
(is a) prefix (of)	$v \preccurlyeq w$	$\exists u, u \in \Sigma^* : vu = w$
(is a) suffix (of)	$v \succcurlyeq w$	$\exists u, u \in \Sigma^* : uv = w$
(is a) substring (of)		$\exists x, x \in \Sigma^* : \exists y, y \in \Sigma^*\ xvy = w$

Uniqueness properties:

- $vu_1 = w$ and $vu_2 = w$ implies $u_1 = u_2$
  - unique suffix is denoted as a quotient $u = w/v$
- $u_1 v = w$ and $u_2 v = w$ implies $u_1 = u_2$

Operators and functions

Name	Notation	Definition
length	$\|v\|$	inductive 1. $\|\varepsilon\| = 0$ 2. $\|aw\| = 1 + \|w\|$
count, for all $c \in \Sigma$	$\#_c(v)$	Inductive 1. $\#_c(\varepsilon) = 0$ 2. $\#_c(cw) = 1 + \#_c(w)$ 3. $\#_c(aw) = \#_c(w), a \neq c$
concatenation of strings	juxtaposition wv	inductive 1. $\varepsilon v = v$ 2. $(aw)v = a(wv)$
concatenation of languages	$L_1 \cdot L_2$	set comprehension $= \{w_1 w_2 \| w_1 \in L_1, w_2 \in L_2\}$
Kleene closure	L^*	set comprehension $= \{w_1 w_2 \ldots w_n \| n \geq 0, w_1, w_2 \ldots, w_n \in L\}$

(Algebraic) properties

Name	Notation (algebraic law)	Proof
unit of string concatenation	$\varepsilon v = v = v\varepsilon$	By induction
associativity of string concatenation	$(wv)u = w(vu)$	By induction
additivity of length operator	$\|wv\| = \|w\| + \|v\|$	By induction
additivity of count operator	$\#_c(wv) = \#_c(w) + \#_c(v)$	By induction
zero of language concatenation	$\emptyset \cdot L = \emptyset = L \cdot \emptyset$	Element wise
unit of language concatenation	$\{\varepsilon\} \cdot L = L = L \cdot \{\varepsilon\}$	Element wise