Finite Automata

2IT70 Finite Automata and Process Theory

Technische Universiteit Eindhoven

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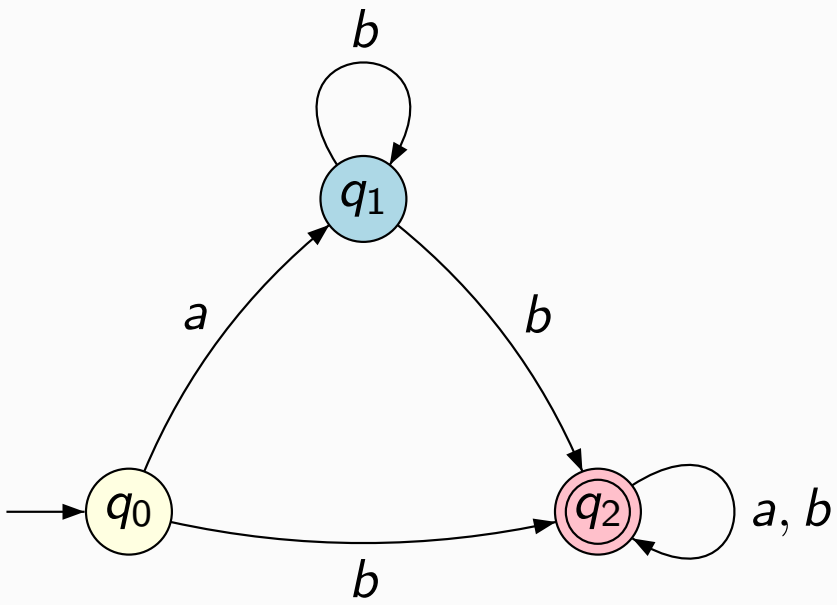
Deterministic finite automaton

DFA $D = (Q, \Sigma, \delta, q_0, F)$

- Q finite set of states
- Σ finite alphabet
- $\delta : Q \times \Sigma \to Q$
  transition function
- $q_0 \in Q$ initial state
- $F \subseteq Q$ set of final states

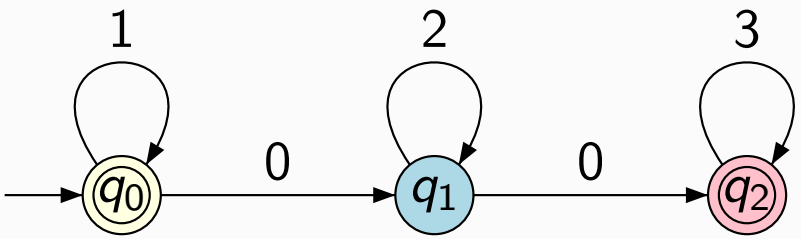


language accepted by DFA D

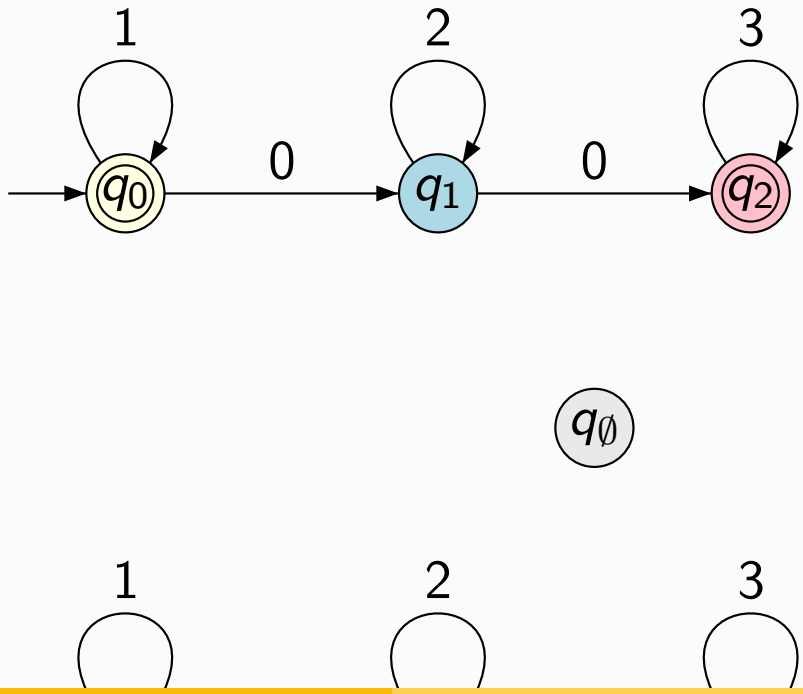
$$\mathcal{L}(D) = \{\, w \in \Sigma^* \mid \exists q \in F : (q_0, w) \vdash_D^* (q, \varepsilon) \,\}$$

A non-DFA

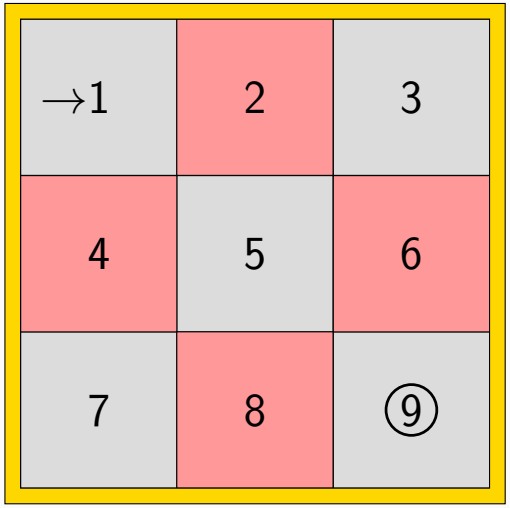
missing transitions



adding a sink state, missing transitions, and self-loops



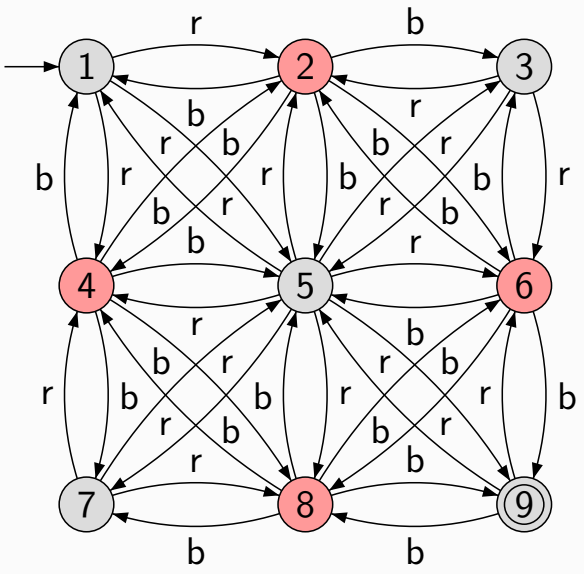
A chess board



	b	r
$\rightarrow 1$	5	$2, 4$
2	$1, 3, 5$	$4, 6$
3	5	$2, 4$
4	$1, 5, 7$	$2, 8$
5	$1, 3, 7, 9$	$2, 4, 6, 8$
6	$3, 5, 9$	$2, 8$
7	5	$4, 8$
8	$5, 7, 9$	$4, 6$
9	5	$6, 8$

$s \xrightarrow{b} t$ iff square s is touching the black square t
$s \xrightarrow{r} t$ iff square s is touching the red square t

Clicker question 2.1



(I) All strings b^{2n} for $n \geqslant 1$ are accepted

(II) No string r^n for $n \geqslant 0$ is accepted

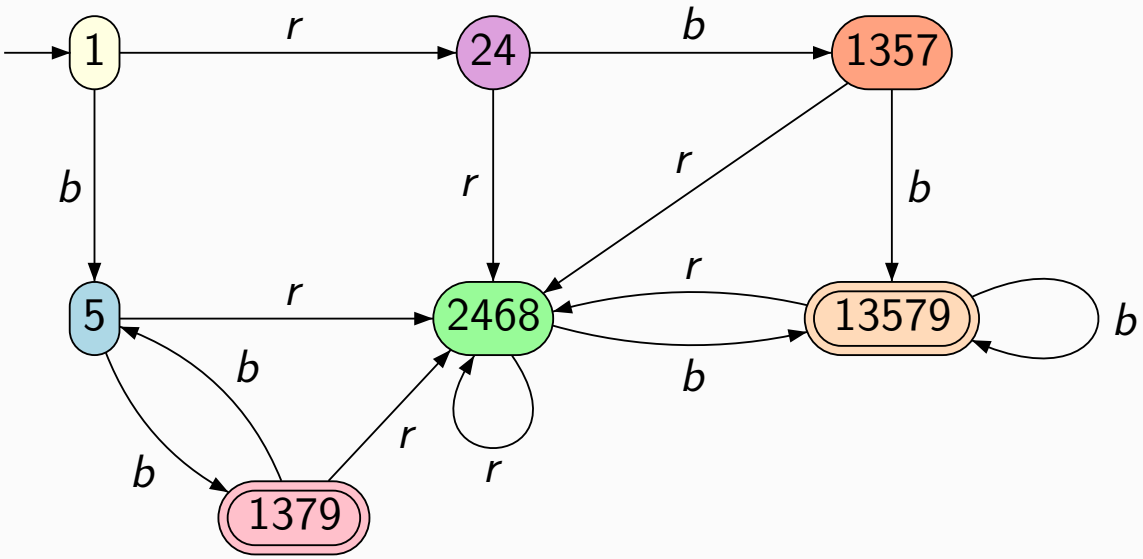
A. Both (I) and (II) are true

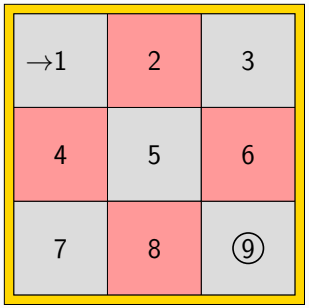
B. Only (I) is true

C. Only (II) is true

D. Both (I) and (II) are false

Chess board: equivalent DFA





(I) All strings b^{2n} for $n \geqslant 1$ are accepted

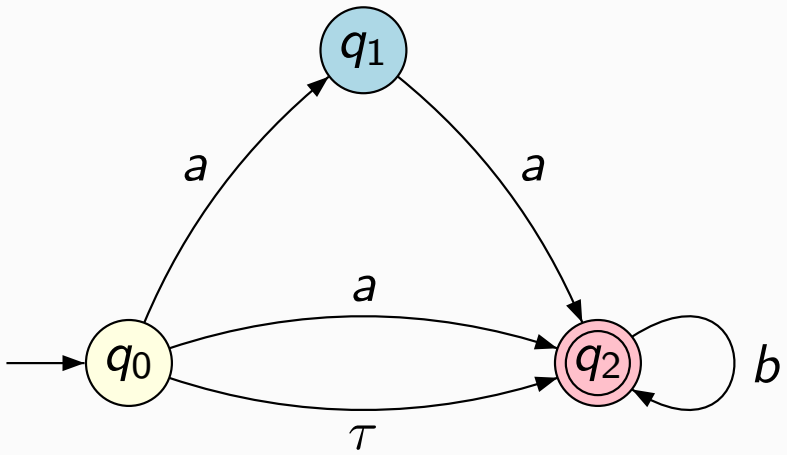
(II) No string r^n for $n \geqslant 0$ is accepted

Non-deterministic finite automaton with τ-moves

silent action $\tau \notin \Sigma$

NFA $N = (Q, \Sigma, \rightarrow, q_0, F)$

- Q finite set of states
- Σ finite alphabet
- $\rightarrow : Q \times \Sigma_\tau \times Q$
  transition relation
- $q_0 \in Q$ initial state
- $F \subseteq Q$ set of final states

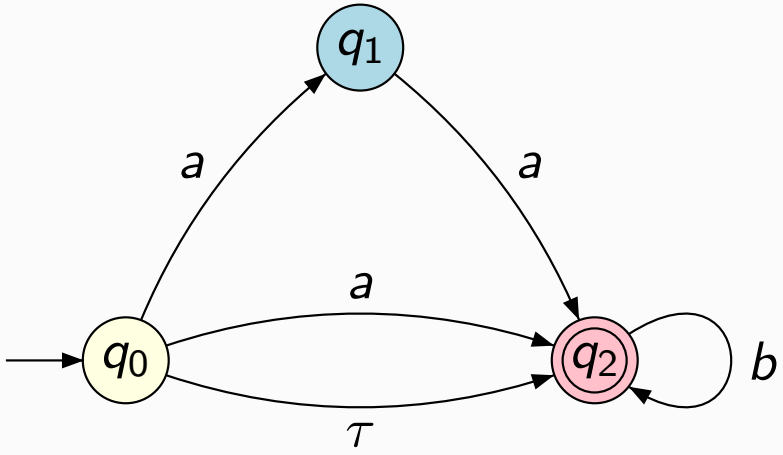


language accepted by NFA N

$$\mathcal{L}(N) = \{\, w \in \Sigma^* \mid \exists q \in F : (q_0, w) \vdash_N^* (q, \varepsilon) \,\}$$

Clicker question 2.2

What language is accepted by the following NFA?



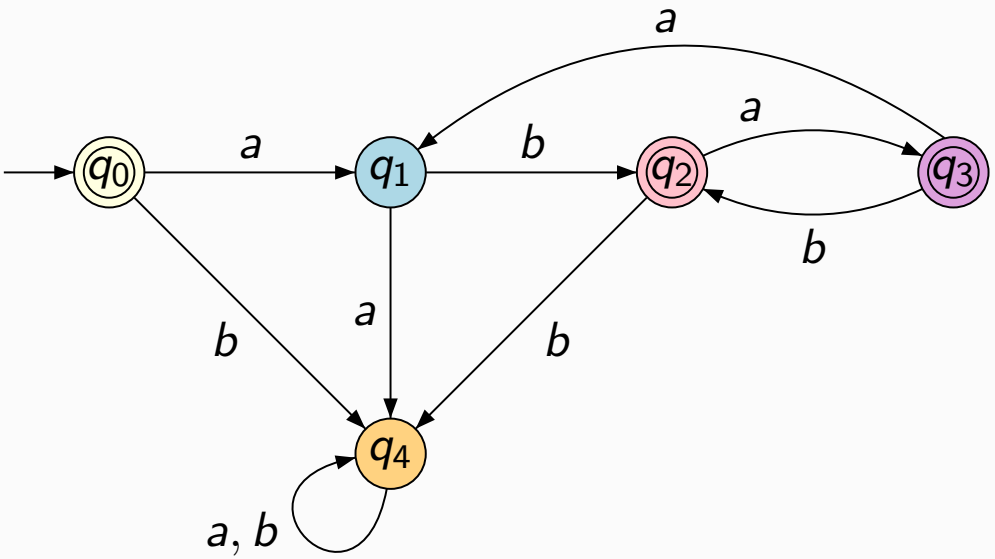
A. $\{ ab^n \mid n \geqslant 1 \}$

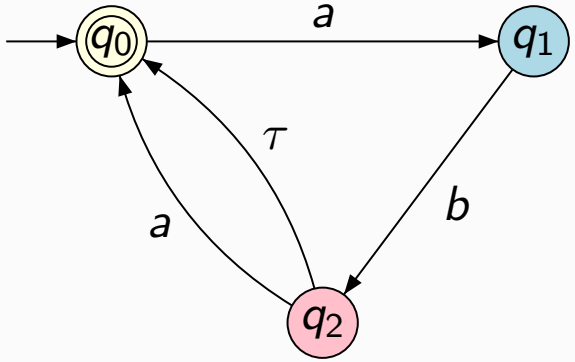
B. $\{ ab^n \mid n \geqslant 0 \}$

C. $\emptyset$

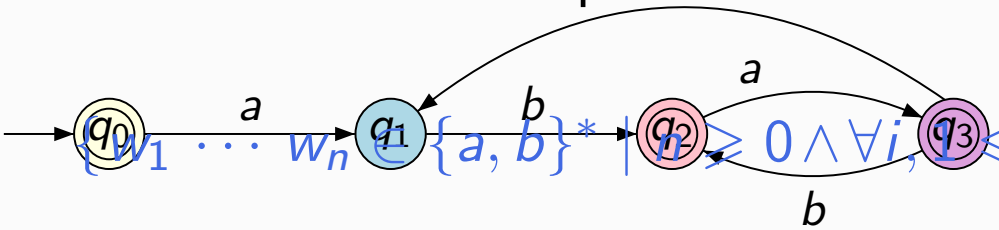
D. $\{ b^n \mid n \geqslant 0 \} \cup \{ ab^n \mid n \geqslant 0 \} \cup \{ aab^n \mid n \geqslant 0 \}$

A DFA vs. an NFA

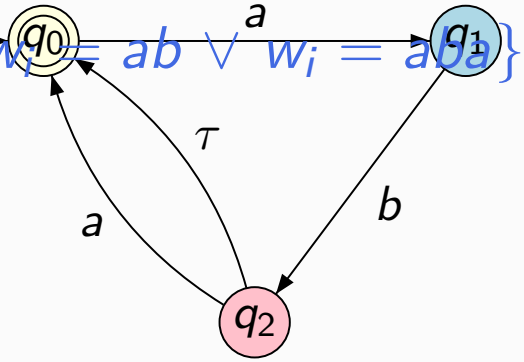




both automata accept a



$\{w_1 \cdots w_n \in \{a, b\}^* \mid n \geqslant 0 \wedge \forall i, 1 \leqslant i \leqslant n\colon w_i = ab \vee w_i = aba\}$



which is also $(ab + aba)^*$

Two theorems

THEOREM

if $L = \mathcal{L}(D)$ for some DFA, then $L = \mathcal{L}(N)$ for some NFA

PROOF say $D = (Q, \Sigma, \delta, q_0, F)$

put $N = (Q, \Sigma, \rightarrow_N, q_0, F)$ with $\delta(q, a) = q' \iff q \xrightarrow{a}_N q'$

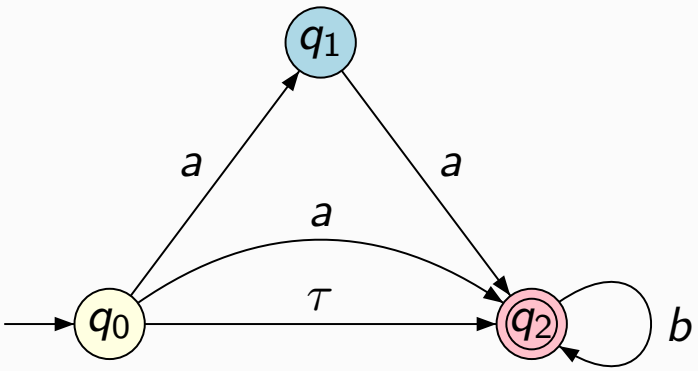
then $(q, w) \vdash_D (q', w')$ iff $(q, w) \vdash_N (q', w')$

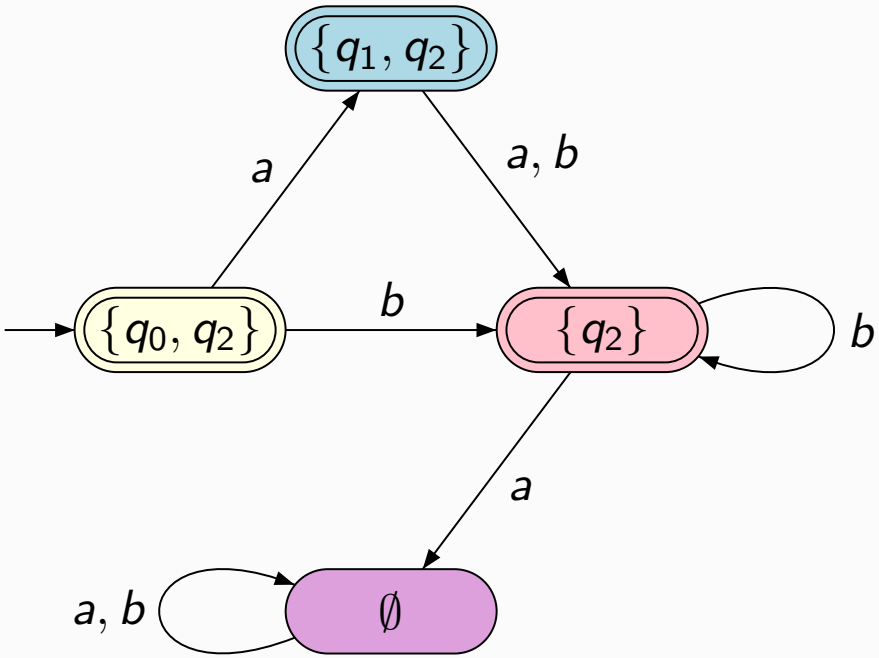
thus $\mathcal{L}(N) = \mathcal{L}(D)$ $\square$

Two theorems (cont.)

THEOREM

if $L = \mathcal{L}(N)$ for some NFA, then $L = \mathcal{L}(D)$ for some DFA





ε-closure $E(q_0) = \{q_0, q_2\}$

Two theorems (cont.)

LEMMA for every NFA N

$$(q, w) \vdash_N^* (q', w') \quad \text{iff} \quad (q, wv) \vdash_N^* (q', w'v)$$

Two theorems (cont.)

PROOF say $N = (Q_N, \Sigma, \rightarrow_N, q_N^0, F_N)$

ε-closure $E(q) = \{ \bar{q} \in Q_N \mid (q, \varepsilon) \vdash_N^* (\bar{q}, \varepsilon) \}$ for $q \in Q_N$

put $D = (Q_D, \Sigma, \delta_D, q_D^0, F_D)$ with

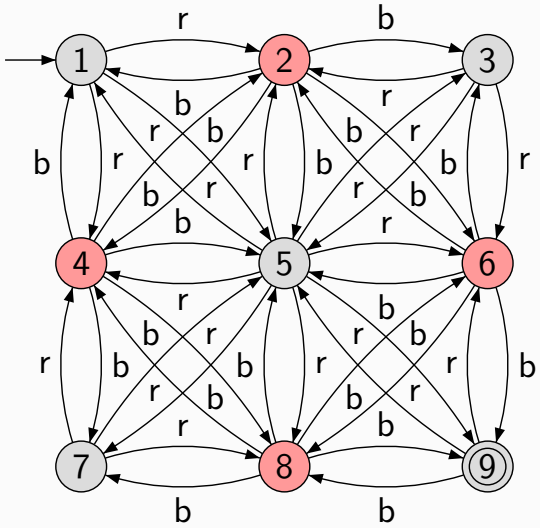
- set of states $Q_D = \mathcal{P}(Q_N)$
- transition function $\delta_D(Q, a) = \bigcup \{ E(\bar{q}) \mid q \in Q,\ q \xrightarrow{a}_N \bar{q} \}$
- initial state $q_D^0 = E(q_N^0)$
- final states $F_D = \{ Q \subseteq Q_N \mid Q \cap F_N \neq \emptyset \}$

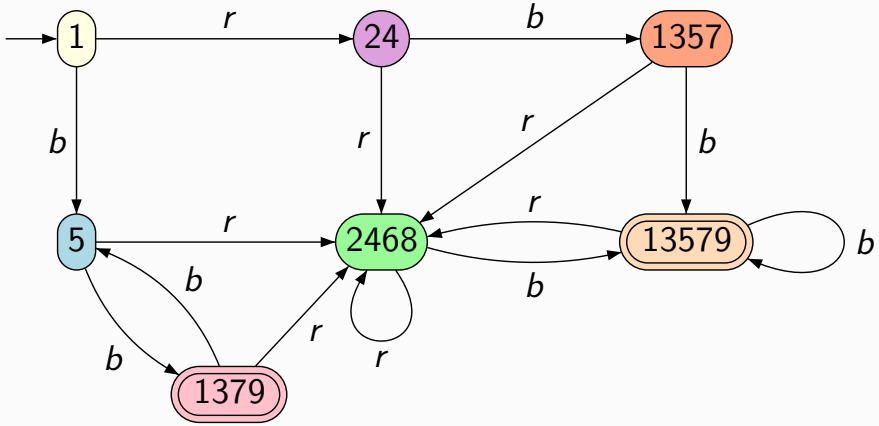
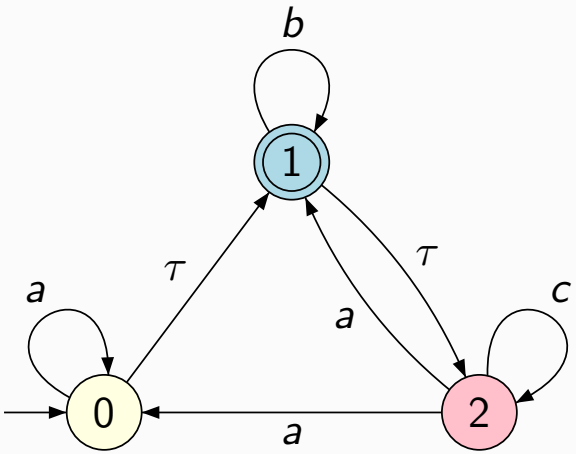
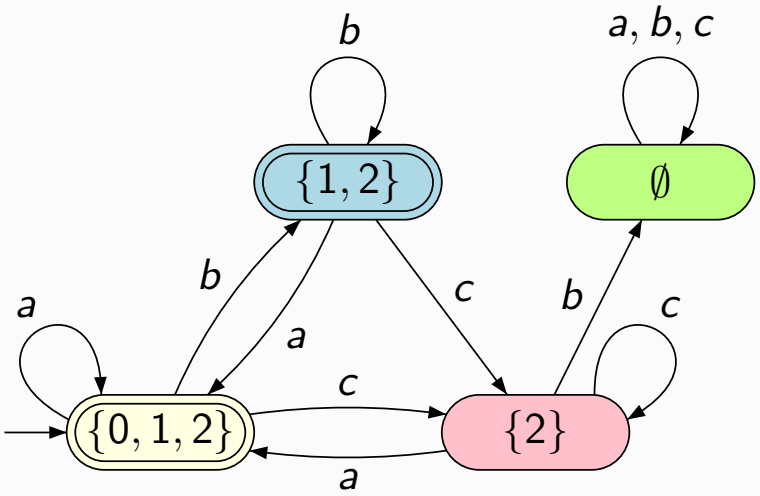
it holds that $(q, w) \vdash_N^* (q', \varepsilon)$ iff

$\exists Q' \subseteq Q_N \colon (E(q), w) \vdash_D^* (Q', \varepsilon)$ and $q' \in Q'$

then $\mathcal{L}(N) = \mathcal{L}(D)$ follows $\square$

An NFA-to-DFA example





Another NFA-to-DFA example