- approximation algorithm of factor α

minimalization problem Π

optimization function *opt* $:\Pi \to \mathbb{N}$

polynomial approximation *appr* $:\Pi \to \mathbb{N}$

α-approximation *appr*, for $\alpha > 1$, satisfies

$$\mathit{opt}(I) \leq \mathit{appr}(I) \leq \alpha \cdot \mathit{opt}(I)$$

for any instance I of Π

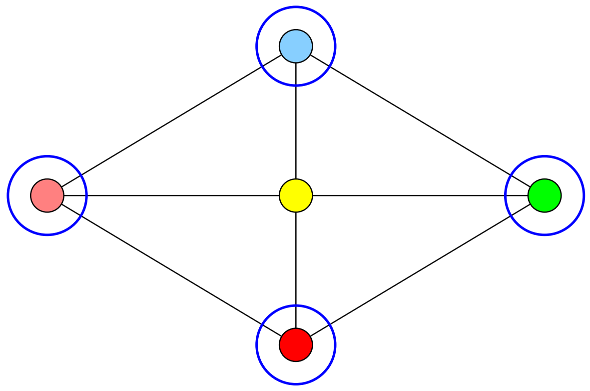
- vertex cover

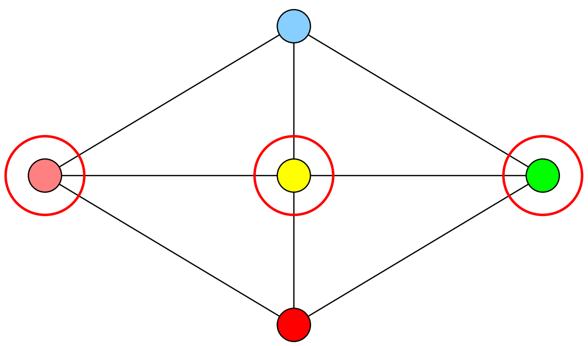
graph $G = (V, E)$

set of nodes $C \subseteq V$ is vertex cover of G

if every edge has an endpoint in C

a minimal vertex cover has minimal number of elements



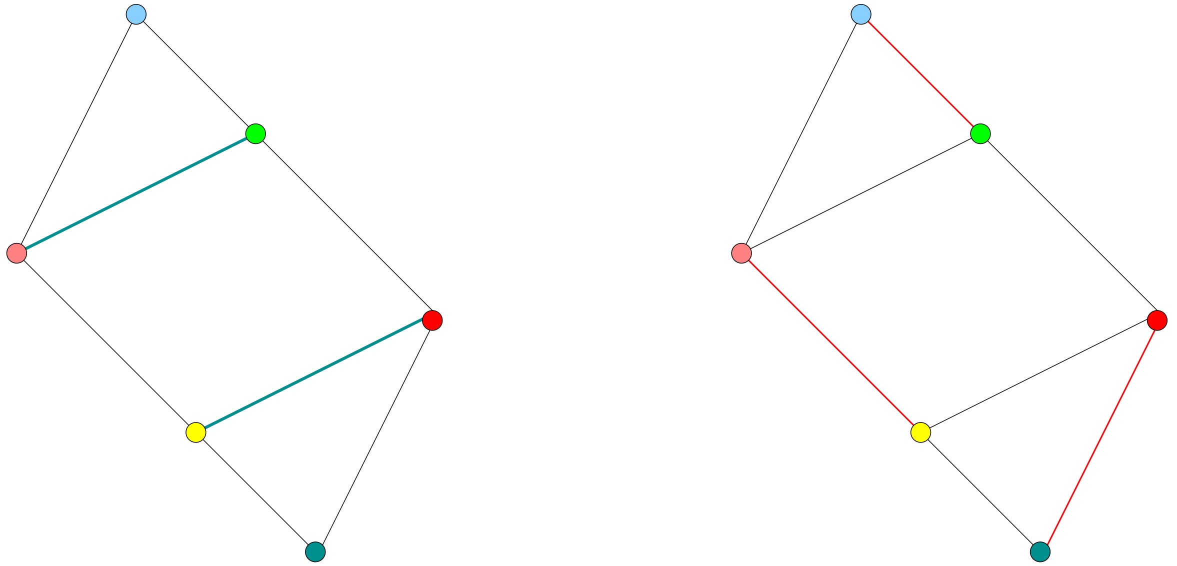


- finding a lower bound for vertex cover

$M \subseteq E$ is a matching

if no two different edges share an endpoint

a maximal matching cannot be extended

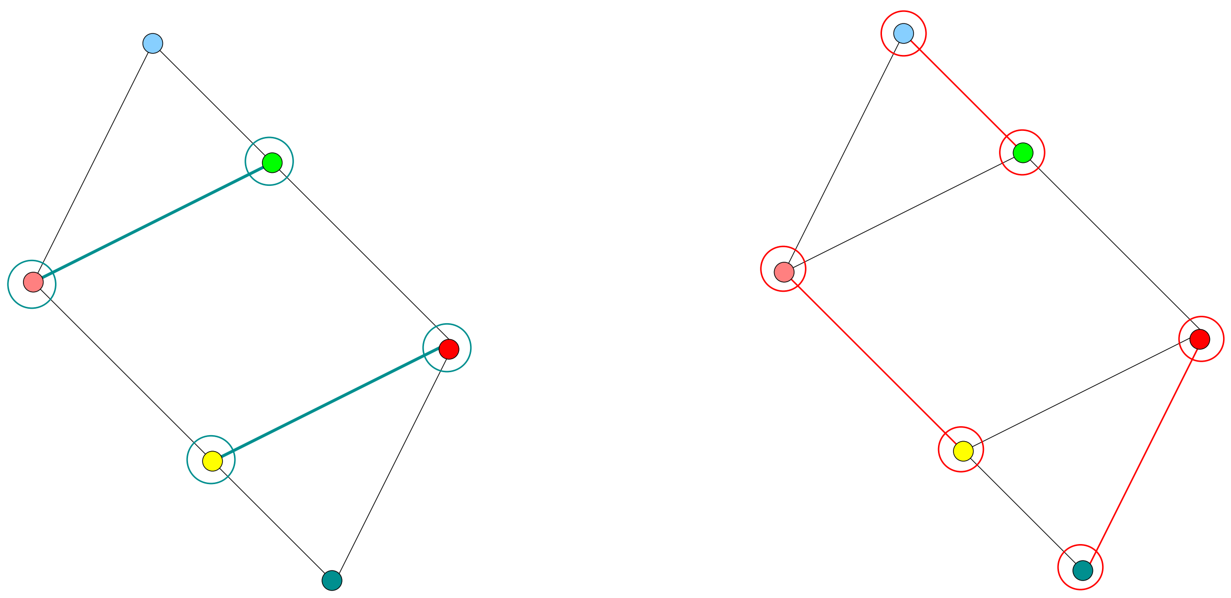


- finding a lower bound for vertex cover

$M \subseteq E$ is a matching

if no two different edges share an endpoint

the nodes of a maximal matching form a vertex cover

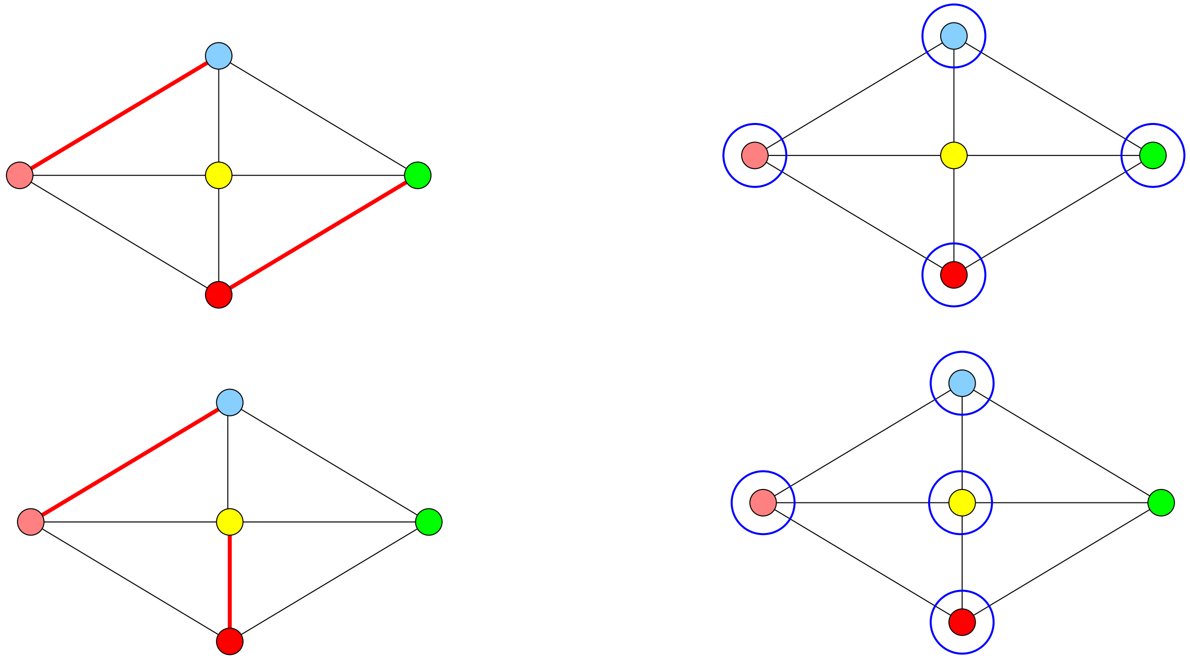


- 2-approximiation for vertex cover

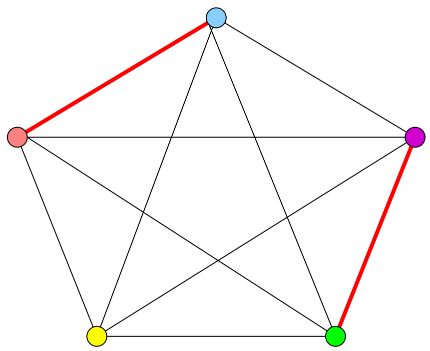
  compute maximal matching M for G, output set of matched vertices

- theorem $\#M \leq OPT \leq 2 \cdot \#M$

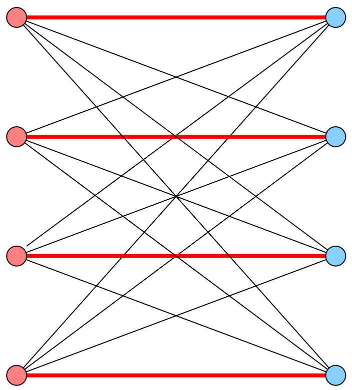
  hence the size of the set of matched vertices is 2-approximation



- tight bounds



$opt = 4$, $appr = 4$



$opt = 4$, $appr = 8$

- bin packing

  set of items $S = \{\ s_1, s_2, \ldots, s_n\ \} \subseteq \mathbb{N}$

  fixed number of bins or sufficiently many bins of size B

  decision problem NP-complete

  SUBSET-SUM $\leq_P$ SET-SPLITTING $\leq_P$ BIN-PACKING

  approximations for decision problem

  next_fit if factor 2, *decreasing_first_fit* of factor $\frac{3}{2}$

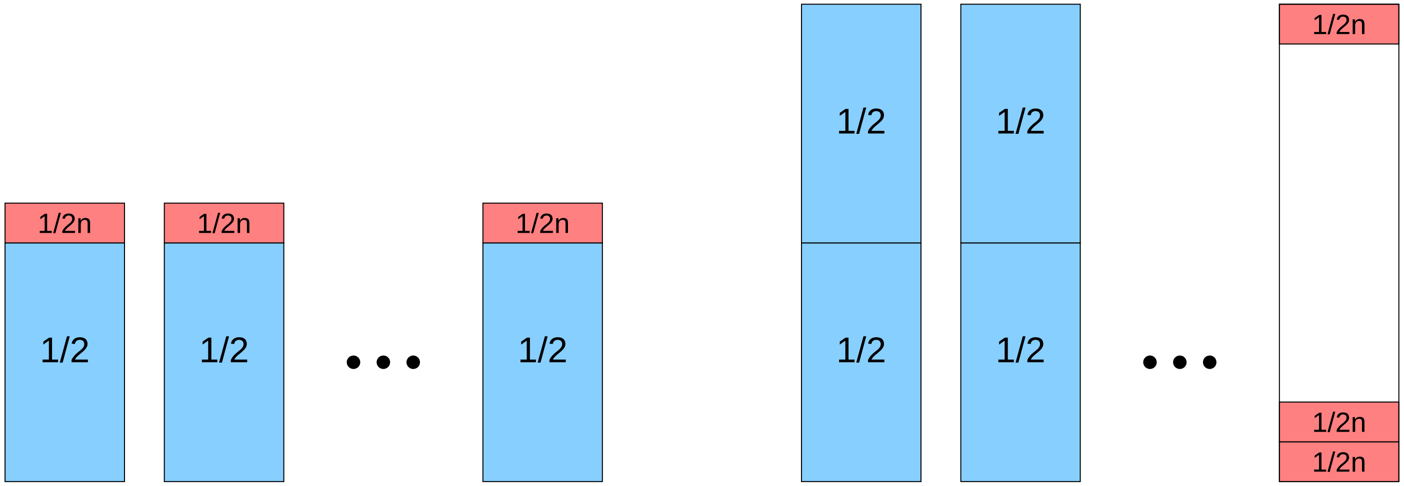
- approximation next_fit

```
k=1;
for i=1 to n do
    if s_i fits in bin_k then
        add s_i to bin_k
    else
        close bin_k;
        k=k+1;
        add s_i to bin_k
    end;
return k
```

- next_fit is 2-approximation for BIN_PACKING analysis for next_fit is tight

items $\{\frac{1}{2}, \frac{1}{2n}, \frac{1}{2}, \frac{1}{2n}, \ldots\}$, bin size 1

$2n$ items of size $\frac{1}{2}$, $2n$ items of size $\frac{1}{2n}$



appr $= 2n$ *opt* $= n + 1$

- approximation decreasing_first_fit

```
// s_1, ..., s_n in descending order
for i=1 to n do
    for j=1 to n do
        if s_i fits in bin_j then
            add s_i to bin_j
    end
end;
return number_of_used_bins
```

- decreasing_first_fit is 3/2-approximation for BIN_PACKING

- traveling salesman optimization problem

  complete graph G

  non-negative edge cost function *cost*

  find minimum cost Hamiltonian cycle

- theorem TSP cannot be approximated unless $P = NP$

  reduction of NP-complete Hamiltonian cycle problem

  edges have distance 1, non-edges have distance $n \cdot \alpha$

  appr $< n \cdot \alpha$ iff Hamiltonian cycle exists

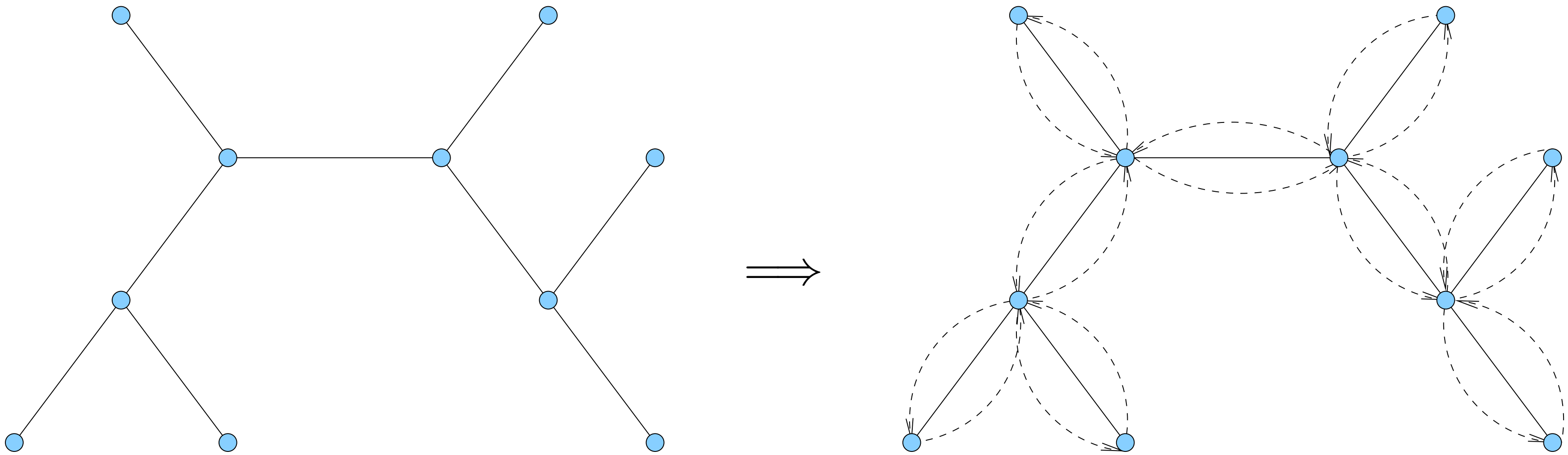
- metric traveling salesman optimization problem

  cost function fulfills triangle inequality, i.e.

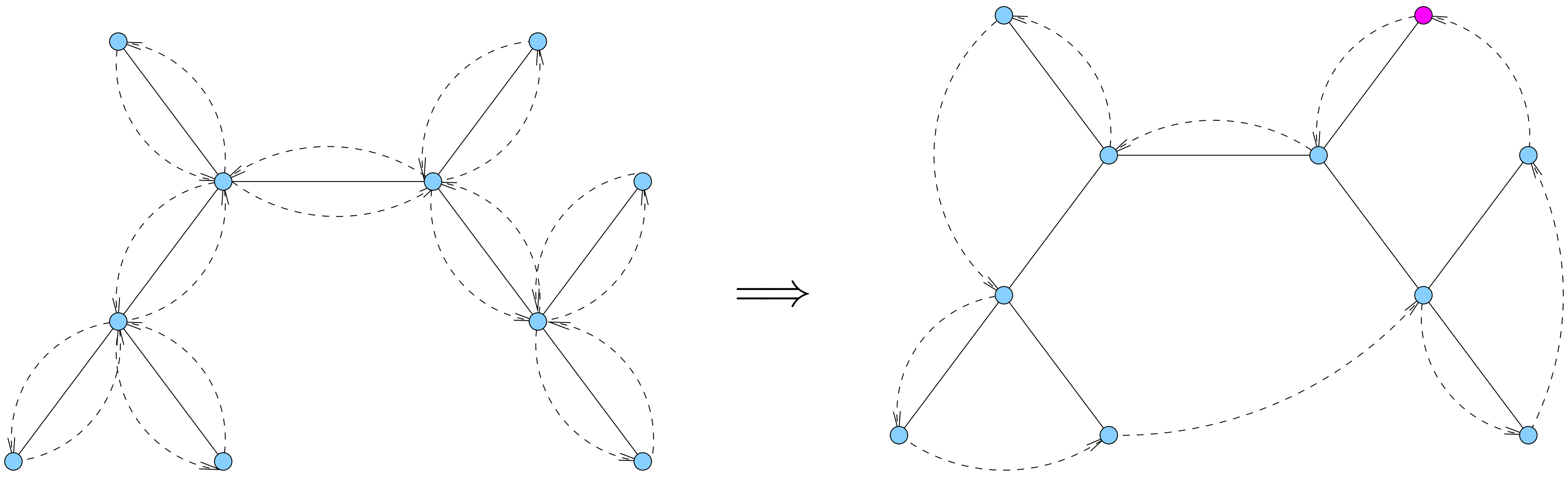
  $\textit{cost}\,(u, w) \leq \textit{cost}\,(u, v) + \textit{cost}\,(v, w)$

- factor 2 approximation algorithm
  - find minimal spanning tree for G
  - double every edge
  - find the Euler tour
  - output the tour skipping doubles

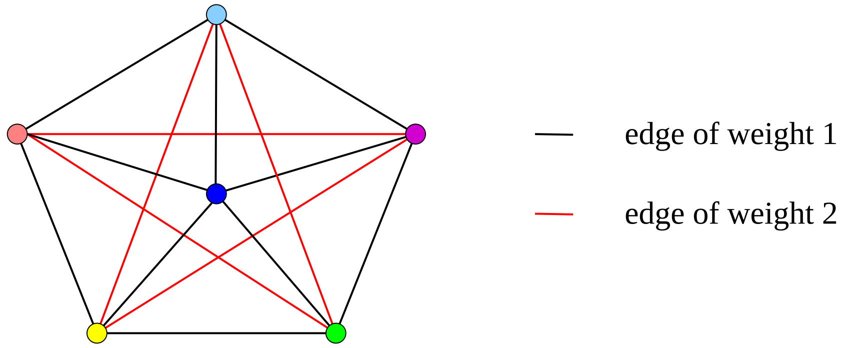
Euler path on minimal spanning tree for G

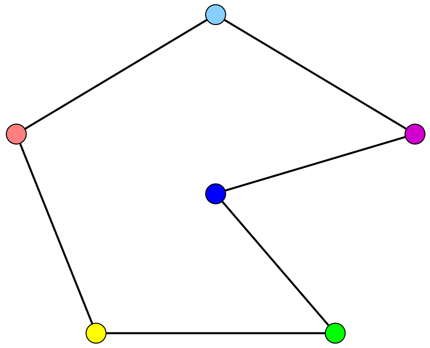


skip doubles to obtain Hamilton tour

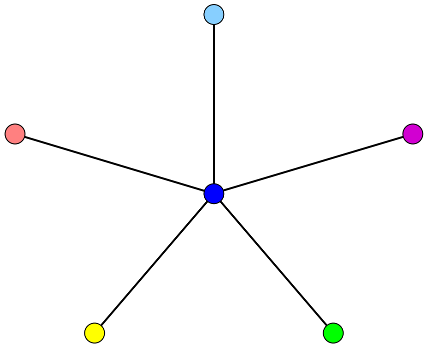


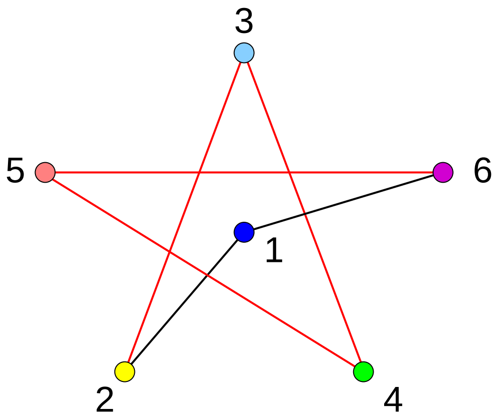
- tightness of the lowerbound





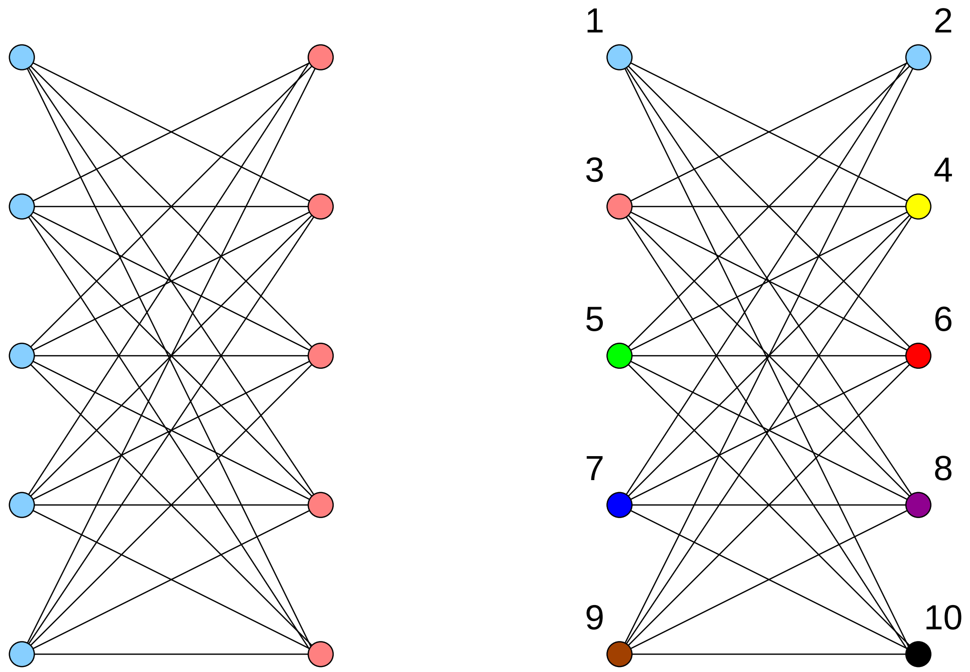
$opt = 5$





$appr = 10$

- greedy coloring algorithm for graph coloring



- if $P \neq NP$ then no better than 4/3-approximation