Exam Basic Mathematics, 2DL03, Wednesday 12th of March 2008, 9.00–12.00.

If you don't have an identity number, write on the first paper behind **Ident.nr:** NONE.

The exam consists of two parts, a **Common part** and a **Pre-master part/Part doorstroomminor HBO**. The **Common part** has to be done in two hours and contains 9 problems, for which you can get 40 points. The **Pre-master part/Part doorstroomminor HBO** has to be done in one hour and contains 5 problems, for which you can get 20 points. For information about the partition of the points over the exercises, see at the end. If you are making both parts, you get two marks, one for the first part and one for the entire exam.

All students have to make the **Common part**.
HBO-students, which want to do the minor Academic orientation, only have to do the first part. These students have three hours to make the exam.
All other students have also to do the **Pre-master part/Part doorstroomminor HBO**.
For HAN-students: the **Common part** is level 3 and the whole examination is level 4.

Formulate the computations and the results of the problems in a clear way.

The use of laptop, graphical and/or programmable calculator, formula chart, book or written sources is not allowed.

Common part

1. Consider the function f with $f(x) = \dfrac{1}{x^2 - 4}$.

   Give a sketch of the graph of the function f.
   Determine the domain $D(f)$ and the range $R(f)$ of the function f.

2. Sketch the set of points (x, y) in $\mathbb{R}^2$ which satisfy the inequalities $x^2 + y^2 > 2y$ and $y > x^2 - 4x$.

3. The angle ϑ is in the interval $(-\pi, 0)$ and $\tan(\vartheta) = 4$.

   Evaluate $\sin(\vartheta)$ and $\cos(\vartheta)$.

4. Find the limit $\lim\limits_{x \to 2} \dfrac{|x - 3| - x^2 - 4x - 3}{x}$.

5. Differentiate the expression $\dfrac{2+\sqrt{x}}{2-\sqrt{x}}$ with respect to x and express the result as a fraction with a simple numerator.

6. Consider the polynomial $p(x) = 2\,x^3 - 3\,x^2 - 32\,x - 15$. One zero of the polynomial is $x = 5$.

   (a) Divide the polynomial $p(x)$ by $(x-5)$ such that the remainder $r(x)$ equals a constant.

   (b) Write the polynomial $p(x)$ as a product of factors of minimal degree.

7. Give the linearization of the function $f(x) = \cos(x)$ about $\dfrac{\pi}{3}$.

8. Evaluate the integral $\displaystyle\int_0^{\pi/2} \dfrac{\sin(x)\,\cos(x)}{\sqrt[3]{\cos^2(x)+1}}\,\mathrm{d}x$.

9. Evaluate the integral $\displaystyle\int_1^{e} x\,\ln(x)\,\mathrm{d}x$.

10. Show that for all x in $\mathbb{R}$ it is true that $4\sin^2(x) - 4\sin^4(x) = \sin^2(2\,x)$.

Pre-master part/Part doorstroomminor HBO

11. Show that for all a and b in $\mathbb{R}$, $a \neq b$, the inequality $\left|\dfrac{\sin^2(b)-\sin^2(a)}{b-a}\right| \leq 1$ is true.

    Hint: use the Mean-Value Theorem.

12. The Taylor polynomial of order 3 of the cosine about the point 0 will be denoted by $p_3(x)$.

    (a) Give the Taylor polynial $p_3(x)$ and approximate $\cos(0.1)$ with the aid of p_3. You may express the approximation as a fraction.

    (b) Let α denote the approximation from part (a).
    Estimate the difference $\cos(0.1) - \alpha$.
    Has the difference $\cos(0.1) - \alpha$ a positive sign?

13. Simplify the expression $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\displaystyle\int_0^{\sqrt{x}} f(t)\mathrm{d}t\right)$.

see next page

14. Simplify $\tan\left(-\arcsin\left(\dfrac{1}{4}\right)\right)$.

15. Show that $\dfrac{16}{3} \leq \displaystyle\int_0^4 \sqrt{x + \sqrt{x}}\,\mathrm{d}x$.

    Give, if necessary, another lower limit for the integral at the right side.

For the problems you can get the following points:

Common part

Problem	1:	4	points	Problem	5:	4	points	Problem	8:	4	points
Problem	2:	4	points	Problem	6a:	2	points	Problem	9:	4	points
Problem	3:	4	points	Problem	6b:	2	points	Problem	10:	4	points
Problem	4:	4	points	Problem	7:	4	points				

The mark for the **Common part** is obtained as follows: the total of the scored points is divided by 4 and is rounded off to the closest natural number.

Pre-master part/Part doorstroomminor HBO

Problem	11:	4	points	Problem	12b:	2	points	Problem	14:	4	points
Problem	12a:	2	points	Problem	13:	4	points	Problem	15:	4	points

The mark for the whole exam, is obtained as follows: the total of the scored points of both parts is divided by 6 and is rounded off to the closest natural number.