EINDHOVEN University of Technology
Department of Mathematics and Computer Science

Examination Basic Mathematics, 2DL03, Wednesday 1 October 2008, 9.00–12.00.

Write clearly the program (Pre-master program or HBO-minor) you are following on the first page of your work.
If you don't have an identity number, write on the first paper behind **Ident.nr:** NONE.

The exam consists of two parts, a **Common part** and a **Pre-master part/Part doorstroomminor HBO**.
The **Common part** has to be done in two hours and contains 9 problems, for which you can get 40 points.
The **Pre-master part/Part doorstroomminor HBO** has to be done in one hour and contains 5 problems, for which you can get 20 points.
For information about the partition of the points over the exercises, see at the end.
If you are making both parts, you get two marks, one for the first part and one for the entire exam.

All students have to make the **Common part**.
HBO-students, which want to do the minor Academic orientation, only have to do the first part. These students have three hours to make the exam.
All other students have also to do the **Pre-master part/Part doorstroomminor HBO**.
For HAN-students: the **Common part** is level 3 and the whole examination is level 4.

Formulate the computations and the results of the exercises in a clear way.

It is not allowed to use a laptop, graphical calculator or chart with formulas.
It is not allowed to use a book or other hand-written material.

Common part

1. Determine all x in $\mathbb{R}$ for which the following inequality holds $\dfrac{x+1}{x+3}+\dfrac{x+3}{x+1}\leq\dfrac{10}{3}$.
2. Find the Taylor polynomial of order four about $x=1$ for $f(x)=x\cdot\ln x$.
   Give the equation of the tangent line to the graph of f through the point (1,0).
3. Consider the function f with $f(x)=\dfrac{2x-1}{x-2}$.
   Determine and simplify $(f\circ f)(x)=f(f(x))$ and find the range of $(f\circ f)$.

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4. Let $\vartheta \in (\frac{1}{2}\pi, \pi)$ and $\tan(\vartheta) = -3$.
   Compute $\sin(\vartheta)$ en $\sin(2\vartheta)$.

5. Find the equation of the tangent line to the curve given by $x^3y^2 = 3y - 2x$ at the point P(1,1).

6. a)Compute $\displaystyle\int_{-1}^{1} (6t^2 + t - 3)\,\mathrm{d}t$.

   b)What is the mean value of $f(t) = 6t^2 + t - 3$ on the interval [-1,1]?
   Find the point(s) at which this mean value is attained.

7. Compute $\displaystyle\int \frac{(1 + \tan(x))^2}{\cos^2(x)}\,\mathrm{d}x$.

8. Simplify $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^{x^3} \frac{t^2}{1+t^6}\,\mathrm{d}t\right)$.

9. Compute $\displaystyle\int (x^2 + x)\ln(x)\mathrm{d}x$.

Pre-master part/Part doorstroomminor HBO

10. Find a such that $x = 5$ is a zero of the polynomial $p(x) = x^3 - 7x^2 + 7x + a$.
    Provide the other zeros for this particular value of a as well.

11. Show that $\sqrt[3]{x+1} - 1 < \dfrac{1}{3}x$ for all $x > 0$

12. Consider the function f with $f(x) = \ln(1 + 2x)$.
    The Taylor polynomial of order 2 of f around 0 will be denoted by $p_2(x)$.
    Answer the subparts of this problem without using a calculator.
    (a) Give the Taylor polynomial $p_2(x)$ and approximate $\ln(1.2)$ by using the Taylor polynomial $p_2(x)$.
    (b) Is $\ln(1.2)$ greater or smaller than the approximation of part (a)?
    Give an estimation for the difference.

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13. Compute $\int_0^1 \arctan(x)\,\mathrm{d}x$.

14. Show that $\sin(2\arccos(x)) = 2x\sqrt{1-x^2}$.

For the problems you can get the following points:

Common part

Problem	1:	5 points	Problem	5:	4 points	Problem	8:	4 points
Problem	2:	6 points	Problem	6a:	2 points	Problem	9:	4 points
Problem	3:	4 points	Problem	6b:	3 points			
Problem	4:	4 points	Problem	7:	4 points			

The mark for the **Common part** is obtained as follows: the total of the scored points is divided by 4 and is rounded off to the closest natural number.

Pre-master part/Part doorstroomminor HBO

Problem	10:	4 points	Problem	12a:	3 points	Problem	13:	4 points
Problem	11:	4 points	Problem	12b:	2 points	Problem	14:	3 points

The mark for the whole exam is obtained as follows: the total of the scored points of both parts is divided by 6 and is rounded off to the closest natural number.

Integral Table

$g(x)$	$\int g(x)\mathrm{d}x$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln(\|x\|)$
$\frac{f'(x)}{f(x)}$	$\ln(\|f(x)\|)$
e^x	e^x
$a^x, a > 0, a \neq 1$	$\frac{a^x}{\ln(a)}$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\frac{1}{\sin^2(x)}$	$-\cot(x)$
$\frac{1}{\cos^2(x)}$	$\tan(x)$
$\tan(x)$	$-\ln(\|\cos(x)\|)$
$\frac{1}{\sin(x)}$	$\ln(\|\tan(\frac{x}{2})\|)$
$\frac{1}{\cos(x)}$	$\ln(\|\tan(\frac{x}{2}+\frac{\pi}{4})\|)$
$e^{ax}\sin(bx), a^2+b^2>0$	$\frac{e^{ax}}{a^2+b^2}\,(a\sin(bx)-b\cos(bx))$
$e^{ax}\cos(bx), a^2+b^2>0$	$\frac{e^{ax}}{a^2+b^2}\,(a\cos(bx)+b\sin(bx))$
$\frac{1}{a^2+x^2}, a>0$	$\frac{1}{a}\ \arctan(\frac{x}{a})$
$\frac{1}{a^2-x^2}, a>0$	$\frac{1}{2a}\ \ln(\left\|\frac{a+x}{a-x}\right\|)$
$\frac{1}{\sqrt{a^2-x^2}}, a>0$	$\arcsin(\frac{x}{a})$
$\frac{1}{\sqrt{a^2+x^2}}, a>0$	$\ln(x+\sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}, a>0$	$\ln(\left\|x+\sqrt{x^2-a^2}\right\|)$
$\sqrt{a^2-x^2}, a>0$	$\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\ \arcsin(\frac{x}{a})$
$\sqrt{a^2+x^2}, a>0$	$\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\ \ln(x+\sqrt{x^2+a^2})$
$\sqrt{x^2-a^2}, a>0$	$\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\ \ln(\left\|x+\sqrt{x^2-a^2}\right\|)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\ln(\cosh(x))$

Remarks

All parameters are real numbers.
The constants of integration have been omitted.

Taylor Polynomials

Function	Taylor polynomial plus O-term
e^x	$1 + x + \frac{1}{2}x^2 + \cdots + \frac{1}{n!}x^n + O(x^{n+1})$
$\cos(x)$	$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots + \frac{(-1)^n}{(2n)!}x^{2n} + O(x^{2n+1})$
$\sin(x)$	$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + O(x^{2n+2})$
$\frac{1}{1+x}$	$1 - x + x^2 + \cdots + (-1)^n x^n + O(x^{n+1})$
$\ln(1+x)$	$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots + \frac{(-1)^n}{n+1}x^{n+1} + O(x^{n+2})$
$\frac{1}{1+x^2}$	$1 - x^2 + x^4 + \cdots + (-1)^n x^{2n} + O(x^{2n+1})$
$\arctan(x)$	$x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots + \frac{(-1)^n}{(2n+1)}x^{2n+1} + O(x^{2n+2})$
$(1+x)^\alpha$	$1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \cdots + \binom{\alpha}{n}x^n + O(x^{n+1})$

- All Taylor polynomials are polynomials around the point 0.
- The binomial coefficients are defined by

$$\binom{\alpha}{k} = \frac{\alpha \cdot (\alpha-1) \cdot (\alpha-2) \cdots (\alpha-(k-1))}{1 \cdot 2 \cdot 3 \cdots k}, \; k = 1, 2, 3, \ldots$$

$$\binom{\alpha}{0} = 1$$