EINDHOVEN University of Technology
Department of Mathematics and Computer Science

Examination Basic Mathematics, 2DL03, Wednesday 20 Januari 2010, 9.00–12.00.

Write clearly the program (Pre-master program or TU/e-minor) you are following on the first page of your work.

Formulate the computations and the results of the exercises in a clear way.

It is not allowed to use a laptop, graphical calculator or chart with formulas.
It is not allowed to use a book or other hand-written material.

---

1. Determine all x in $\mathbb{R}$ for which the following inequality holds $\dfrac{x+2}{x+3} + \dfrac{x+1}{2x+3} \leq 1.$

2. Consider te function f with $f(x) = 2e^{3x} - 1$.

   (a) Solve $f'(x) = 12$.

   (b) Evaluate $f^{-1}(x)$.

3. Consider the function f with the Taylor polynomial of order 3 around 5 given by $p_3(x) = 10 + 3(x-5)^2 + 5(x-5)^3$.

   (a) Find $f(5), f'(5), f''(5), f^{(3)}(5)$ and approximate $f(5.1)$.

   (b) If $0 < f^{(4)}(x) \leq 2$ on the intervall $[5\,,\ 5.1]$, give an estimation of the error $E_3(5.1)$. It is known that $f(5.1) = p_3(5.1) + E_3(5.1)$.

4. Find the Taylor polynomial of order two about $x = 4$ for $f(x) = x^2\sqrt{x}$.
   Give the equation of the line tangent to the graph of f at the point $(4, 32)$.

5. Find the equation of the tangent to the curve given by $x^2y + xy^2 = 6$ at the point P(2,1).

6. Let $\vartheta \in (-\frac{1}{2}\pi, 0)$ and $\tan(\vartheta) = -\frac{2}{3}$.

   Compute $\cos(\vartheta)$ en $\cos(2\vartheta)$.

see next page

7. Prove that $\frac{b-a}{5} \leq \ln(\frac{b}{a}) \leq \frac{b-a}{3}$ for all a and b with $3 \leq a \leq b \leq 5$.

   Apply the Mean-Value Theorem!

8. Find the equations of the two lines tangent to the graph of $y = \arcsin(x)$ with slope 2.

9. Find the limits:

   (a) $\lim\limits_{x \uparrow 5} \dfrac{|x-5|}{2x-10}$.

   (b) $\lim\limits_{x \to 1} \dfrac{x^3+5x^2-6}{x-1}$.

10. (a) Compute $\displaystyle\int_0^{\frac{1}{4}\pi} \cos(2t)\,\mathrm{d}t$.

    (b) What is the mean value of $f(t) = \cos(2t)$ on the intervall $[0, \frac{1}{4}\pi]$?

11. Find $\displaystyle\int x \cdot e^{4x}\,\mathrm{d}x$.

12. Consider the function F with $F(x) = \int_0^{2x} (t-1)^3 \cdot e^{t^2}\,\mathrm{d}t$.

    (a) Find $F'(x)$.

    (b) Find all x with $F'(x) = 0$?

13. Find $\displaystyle\int \frac{x}{\cos^2(x)}\mathrm{d}x$.

14. Compute $\displaystyle\int_1^3 \frac{1}{2\sqrt{x}(x+1)}\,\mathrm{d}x$.

see next page

Examination Basic Mathematics, 2DL03, Wednesday 20 Januari 2010, 9.00–12.00

For the problems you can get the following points:

Problem	1 :	4	points	Problem	7 :	4	points	Problem	12b:	1	points
Problem	2a:	2	points	Problem	8 :	4	points	Problem	13 :	4	points
Problem	2b:	3	points	Problem	9a :	2	points	Problem	14 :	4	points
Problem	3a:	3	points	Problem	9b :	3	points				
Problem	3b:	2	points	Problem	10a:	3	points				
Problem	4 :	5	points	Problem	10b:	1	points				
Problem	5 :	4	points	Problem	11 :	4	points				
Problem	6 :	4	points	Problem	12a:	3	points				

The mark for the exam is obtained as follows: the total of the scored points is divided by 6 and is rounded off to the closest natural number.

Integral Table

$g(x)$	$\int g(x)\mathrm{d}x$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln(\lvert x\rvert)$
$\frac{f'(x)}{f(x)}$	$\ln(\lvert f(x)\rvert)$
e^x	e^x
$a^x, a > 0, a \neq 1$	$\frac{a^x}{\ln(a)}$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\frac{1}{\sin^2(x)}$	$-\cot(x)$
$\frac{1}{\cos^2(x)}$	$\tan(x)$
$\tan(x)$	$-\ln(\lvert\cos(x)\rvert)$
$\frac{1}{\sin(x)}$	$\ln(\lvert\tan(\frac{x}{2})\rvert)$
$\frac{1}{\cos(x)}$	$\ln(\lvert\tan(\frac{x}{2} + \frac{\pi}{4})\rvert)$
$e^{ax}\sin(bx), a^2 + b^2 > 0$	$\frac{e^{ax}}{a^2+b^2}\,(a\sin(bx) - b\cos(bx))$
$e^{ax}\cos(bx), a^2 + b^2 > 0$	$\frac{e^{ax}}{a^2+b^2}\,(a\cos(bx) + b\sin(bx))$
$\frac{1}{a^2+x^2}, a > 0$	$\frac{1}{a}\ \arctan(\frac{x}{a})$
$\frac{1}{a^2-x^2}, a > 0$	$\frac{1}{2a}\ \ln(\left\lvert\frac{a+x}{a-x}\right\rvert)$
$\frac{1}{\sqrt{a^2-x^2}}, a > 0$	$\arcsin(\frac{x}{a})$
$\frac{1}{\sqrt{a^2+x^2}}, a > 0$	$\ln(x + \sqrt{x^2 + a^2})$
$\frac{1}{\sqrt{x^2-a^2}}, a > 0$	$\ln(\left\lvert x + \sqrt{x^2 - a^2}\right\rvert)$
$\sqrt{a^2 - x^2}, a > 0$	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\ \arcsin(\frac{x}{a})$
$\sqrt{a^2 + x^2}, a > 0$	$\frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ \ln(x + \sqrt{x^2 + a^2})$
$\sqrt{x^2 - a^2}, a > 0$	$\frac{x}{2}\sqrt{x^2 - a^2} + \frac{a^2}{2}\ \ln(\left\lvert x + \sqrt{x^2 - a^2}\right\rvert)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\ln(\cosh(x))$

Remarks

All parameters are real numbers.
The constants of integration have been omitted.

Taylor Polynomials

Function	Taylor polynomial plus O-term
e^x	$1 + x + \frac{1}{2}\,x^2 + \cdots + \frac{1}{n!}\,x^n + O(x^{n+1})$
$\cos(x)$	$1 - \frac{1}{2}\,x^2 + \frac{1}{24}\,x^4 + \cdots + \frac{(-1)^n}{(2\,n)!}\,x^{2\,n} + O(x^{2\,n+1})$
$\sin(x)$	$x - \frac{1}{6}\,x^3 + \frac{1}{120}\,x^5 + \cdots + \frac{(-1)^n\,x^{2\,n+1}}{(2\,n+1)!} + O(x^{2\,n+2})$
$\frac{1}{1+x}$	$1 - x + x^2 + \cdots + (-1)^n\,x^n + O(x^{n+1})$
$\ln(1+x)$	$x - \frac{1}{2}\,x^2 + \frac{1}{3}\,x^3 + \cdots + \frac{(-1)^n}{n+1}\,x^{n+1} + O(x^{n+2})$
$\frac{1}{1+x^2}$	$1 - x^2 + x^4 + \cdots + (-1)^n\,x^{2\,n} + O(x^{2\,n+1})$
$\arctan(x)$	$x - \frac{1}{3}\,x^3 + \frac{1}{5}\,x^5 + \cdots + \frac{(-1)^n}{(2\,n+1)}\,x^{2\,n+1} + O(x^{2\,n+2})$
$(1+x)^\alpha$	$1 + \left(\begin{array}{c}\alpha\\1\end{array}\right) x + \left(\begin{array}{c}\alpha\\2\end{array}\right) x^2 + \cdots + \left(\begin{array}{c}\alpha\\n\end{array}\right) x^n + O(x^{n+1})$

- All Taylor polynomials are polynomials around the point 0.
- The binomial coefficients are defined by

$$\left(\begin{array}{c}\alpha\\k\end{array}\right) = \frac{\alpha \cdot (\alpha-1) \cdot (\alpha-2) \cdots (\alpha-(k-1))}{1 \cdot 2 \cdot 3 \cdots k}\,,\ k = 1, 2, 3, \ldots$$

$$\left(\begin{array}{c}\alpha\\0\end{array}\right) = 1$$