EINDHOVEN University of Technology
Department of Mathematics and Computer Science

Examination Basic Mathematics, 2DL03, Wednesday 23 June 2010, 9.00–12.00.

Write clearly the program (Pre-master program or TU/e-minor) you are following on the first page of your work.

Formulate the computations and the results of the exercises in a clear way.

It is not allowed to use a laptop, graphical calculator or chart with formulas.
It is not allowed to use a book or other hand-written material.

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1. Consider the two circles given by the equations $x^2 + y^2 - 4y = 0$ and $x^2 - 2x + y^2 + 2y = 0$.
   (a) Find their centre and their radius.
   (b) Compute the distance between the two centres.
   (c) Describe the region defined by the following pair of inequalities $x^2 + y^2 - 4y > 0$ and $x^2 - 2x + y^2 + 2y < 0$.
2. Consider the function f with $f(x) = \sqrt[5]{x^3}$.
   Give the answers without using a calculator.
   (a) Find the linearization of f about $x = 1$.
   (b) Find the approximation of $(1.1)^{\frac{3}{5}}$ using this linearization.
   (c) Is $(1.1)^{\frac{3}{5}}$ larger or less than the approximation in (b)?
   Provide an estimate of the error E(1.1).
3. Solve $e^x + e^{-x} = \frac{5}{2}$.
4. Evaluate the Taylor polynomial of third order about $x = 3$ of $f(x) = x^3-2x^2+3x-4$.
5. Consider the curve k given by the equation $3x^2 + xy + y^2 = 5$.
   (a) Evaluate y'(x).
   (b) Find the equation of the tangent line to the given curve at the point P(1,1).
   Find the equation of the normal line at P as well.
6. Prove the identity $4 - 4\sin^4(x) = \sin^2(2x) + 4\cos^2(x)$.

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7. Show that $\arcsin(x) < x$ for all $x \in [-1, 0)$.

8. Find equations of the two straight lines tangent to the graph of $y = x^3 - x$ having slope 2.

9. Compute the following limits

   (a) $\lim_{x \to 2} \dfrac{3x^2 - 12}{4x - 8}$.

   (b) $\lim_{x \to 1} \dfrac{x^3 - 2x^2 - 8x + 9}{x - 1}$.

10. (a) Compute $\displaystyle\int_{-1}^{2} |x^2 - 1| \; \mathrm{d}x$.

    (b) What is the mean value of $f(x) = |x^2 - 1|$ at the interval [-1,2]? Also find the point(s) at which this mean value is attained.

11. Evaluate $\displaystyle\int \frac{e^x}{(e^x + 1)^4} \, \mathrm{d}x$.

12. Consider the function F with $F(x) = \int_0^{x^2} \frac{t^2 - 1}{t + 2} \; \mathrm{d}t$.

    (a) Evaluate $F'(x)$.

    (b) Find all x with $F'(x) > 0$.

13. Compute $\displaystyle\int_2^7 \frac{x + 7}{\sqrt{x + 2}} \, \mathrm{d}x$.

14. Compute $\displaystyle\int_1^e x\sqrt{x} \cdot \ln(x) \, \mathrm{d}x$.

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For the problems you can get the following points:

Problem	1a:	2	points	Problem	5a:	3	points	Problem	10b:	2	points
Problem	1b:	2	points	Problem	5b:	2	points	Problem	11:	4	points
Problem	1c:	1	points	Problem	6:	4	points	Problem	12a:	3	points
Problem	2a:	2	points	Problem	7:	3	points	Problem	12b:	2	points
Problem	2b:	1	points	Problem	8:	3	points	Problem	13:	4	points
Problem	2c:	2	points	Problem	9a:	2	points	Problem	14:	5	points
Problem	3:	3	points	Problem	9b:	3	points				
Problem	4:	4	points	Problem	10a:	3	points				

The mark for the exam is obtained as follows: the total of the scored points is divided by 6 and is rounded off to the closest natural number.

Integral Table

$g(x)$	$\int g(x)\mathrm{d}x$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln(\lvert x \rvert)$
$\frac{f'(x)}{f(x)}$	$\ln(\lvert f(x) \rvert)$
e^x	e^x
$a^x, a > 0, a \neq 1$	$\frac{a^x}{\ln(a)}$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\frac{1}{\sin^2(x)}$	$-\cot(x)$
$\frac{1}{\cos^2(x)}$	$\tan(x)$
$\tan(x)$	$-\ln(\lvert \cos(x) \rvert)$
$\frac{1}{\sin(x)}$	$\ln(\lvert \tan(\frac{x}{2}) \rvert)$
$\frac{1}{\cos(x)}$	$\ln(\lvert \tan(\frac{x}{2} + \frac{\pi}{4}) \rvert)$
$e^{ax}\sin(bx), a^2 + b^2 > 0$	$\frac{e^{ax}}{a^2+b^2}\,(a\sin(bx) - b\cos(bx))$
$e^{ax}\cos(bx), a^2 + b^2 > 0$	$\frac{e^{ax}}{a^2+b^2}\,(a\cos(bx) + b\sin(bx))$
$\frac{1}{a^2+x^2}, a > 0$	$\frac{1}{a}\ \arctan(\frac{x}{a})$
$\frac{1}{a^2-x^2}, a > 0$	$\frac{1}{2a}\ \ln(\left\lvert \frac{a+x}{a-x} \right\rvert)$
$\frac{1}{\sqrt{a^2-x^2}}, a > 0$	$\arcsin(\frac{x}{a})$
$\frac{1}{\sqrt{a^2+x^2}}, a > 0$	$\ln(x + \sqrt{x^2 + a^2})$
$\frac{1}{\sqrt{x^2-a^2}}, a > 0$	$\ln(\left\lvert x + \sqrt{x^2 - a^2} \right\rvert)$
$\sqrt{a^2 - x^2}, a > 0$	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\ \arcsin(\frac{x}{a})$
$\sqrt{a^2 + x^2}, a > 0$	$\frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ \ln(x + \sqrt{x^2 + a^2})$
$\sqrt{x^2 - a^2}, a > 0$	$\frac{x}{2}\sqrt{x^2 - a^2} + \frac{a^2}{2}\ \ln(\left\lvert x + \sqrt{x^2 - a^2} \right\rvert)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\ln(\cosh(x))$

Remarks

All parameters are real numbers.
The constants of integration have been omitted.

Taylor Polynomials

Function	Taylor polynomial plus O-term
e^x	$1 + x + \frac{1}{2}\, x^2 + \cdots + \frac{1}{n!}\, x^n + O(x^{n+1})$
$\cos(x)$	$1 - \frac{1}{2}\, x^2 + \frac{1}{24}\, x^4 + \cdots + \frac{(-1)^n}{(2\,n)!}\, x^{2\,n} + O(x^{2\,n+1})$
$\sin(x)$	$x - \frac{1}{6}\, x^3 + \frac{1}{120}\, x^5 + \cdots + \frac{(-1)^n\, x^{2\,n+1}}{(2\,n+1)!} + O(x^{2\,n+2})$
$\frac{1}{1+x}$	$1 - x + x^2 + \cdots + (-1)^n\, x^n + O(x^{n+1})$
$\ln(1+x)$	$x - \frac{1}{2}\, x^2 + \frac{1}{3}\, x^3 + \cdots + \frac{(-1)^n}{n+1}\, x^{n+1} + O(x^{n+2})$
$\frac{1}{1+x^2}$	$1 - x^2 + x^4 + \cdots + (-1)^n\, x^{2\,n} + O(x^{2\,n+1})$
$\arctan(x)$	$x - \frac{1}{3}\, x^3 + \frac{1}{5}\, x^5 + \cdots + \frac{(-1)^n}{(2\,n+1)}\, x^{2\,n+1} + O(x^{2\,n+2})$
$(1+x)^\alpha$	$1 + \binom{\alpha}{1}\, x + \binom{\alpha}{2}\, x^2 + \cdots + \binom{\alpha}{n}\, x^n + O(x^{n+1})$

- All Taylor polynomials are polynomials around the point 0.
- The binomial coefficients are defined by

$$\binom{\alpha}{k} = \frac{\alpha \cdot (\alpha-1) \cdot (\alpha-2) \cdots (\alpha-(k-1))}{1 \cdot 2 \cdot 3 \cdots k}\,,\ k = 1, 2, 3, \ldots$$

$$\binom{\alpha}{0} = 1$$