EINDHOVEN University of Technology
Department of Mathematics and Computer Science

Examination Basic Mathematics, 2DL03, Wednesday 19 January 2011, 9.00–12.00.

Write clearly the program (Pre-master program or TU/e-minor) you are following on the first page of your work.

Formulate the computations and the results of the exercises in a clear way.

It is not allowed to use a laptop, graphical calculator or chart with formulas.
It is not allowed to use a book or other printed or hand-written material.

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1. Show that $x = -2$ and $x = 3$ are the only two solutions of the equation $x^4 + x^3 - 6x^2 - 14x - 12 = 0$.

2. Consider the function f with $f(x) = 3e^{-x} + 2$.

   (a) Solve $f'(x) = -6$.

   (b) Determine the inverse function f^{-1}.

3. Consider the function f with $f(x) = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$.
   Give the answers without using a calculator.

   (a) Find the linearization of f about $x = 1$.

   (b) Find the approximation of $(1.1)^{-\frac{1}{3}}$ using this linearization.

   (c) Is $(1.1)^{-\frac{1}{3}}$ greater than or less than the approximation in (b)?
   Give an estimation of the error E(1.1).

4. (a) Find the Taylor polynomial of third order about $x = \pi$ of $f(x) = \sin(x)$.

   (b) Find the Taylor polynomial of third order about $x = 0$ of $g(x) = \frac{1}{2x+4}$.

5. Consider the curve k given by the equation $2\sqrt{y} + y - 2x - 3 = 0$.
   Find the equation of the tangent line to the given curve at the point P(0,1).

6. Let $\theta = \arctan(\frac{5}{12})$.
   Compute $\sin(\theta)$ and $\sin(2\theta)$.

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7. Solve the inequality $\log_3(x-5)+\log_3(20-x)\leq \log_3 2+2\log_3 5$.

8. Show that $\arctan(x)-\arctan(2)<\frac{1}{5}(x-2)$ for all $x>2$.
   Apply the Mean-Value Theorem!

9. Consider the functions f and g given by $f(x)=-2x+12$ and $g(x)=2\sqrt{x}$.
   Show that the graphs of f and g intersect perpendicular.

10. (a) Let $f(x)=\int_0^{x^2}\sqrt{t^3+1}\,\mathrm{d}t$. Find $f'(x)$.
    (b) Let $g(x)=\sin^2(x\sqrt{x})$. Find $g'(x)$.

11. (a) Compute $\displaystyle\int_0^3 |x^2-4|\ \mathrm{d}x$.
    (b) What is the mean value of $f(x)=|x^2-4|$ on the interval [0,3]?
    Also find the point(s) at which this mean value is attained.

12. Evaluate $\displaystyle\int \frac{\ln(x)}{\sqrt{x}}\,\mathrm{d}x$.

13. Evaluate $\displaystyle\int x^3\sqrt{1+x^2}\,\mathrm{d}x$.

14. Compute $\displaystyle\int_{-2}^{2}\arctan\left(\frac{x}{2}\right)\mathrm{d}x$.

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For the problems you can get the following points:

Problem	1:	4	points	Problem	5:	4	points	Problem	11b:	2	points
Problem	2a:	2	points	Problem	6:	4	points	Problem	12:	4	points
Problem	2b:	3	points	Problem	7:	4	points	Problem	13:	4	points
Problem	3a:	2	points	Problem	8:	4	points	Problem	14:	4	points
Problem	3b:	1	points	Problem	9:	4	points				
Problem	3c:	2	points	Problem	10a:	2	points				
Problem	4a:	2	points	Problem	10b:	2	points				
Problem	4b:	3	points	Problem	11a:	3	points				

The mark for the exam is obtained as follows: the total of the scored points is divided by 6 and is rounded off to the closest natural number.

Integral Table

$g(x)$	$\int g(x)\mathrm{d}x$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln(\lvert x\rvert)$
$\frac{f'(x)}{f(x)}$	$\ln(\lvert f(x)\rvert)$
e^x	e^x
$a^x, a > 0, a \neq 1$	$\frac{a^x}{\ln(a)}$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\frac{1}{\sin^2(x)}$	$-\cot(x)$
$\frac{1}{\cos^2(x)}$	$\tan(x)$
$\tan(x)$	$-\ln(\lvert\cos(x)\rvert)$
$\frac{1}{\sin(x)}$	$\ln(\lvert\tan(\frac{x}{2})\rvert)$
$\frac{1}{\cos(x)}$	$\ln(\lvert\tan(\frac{x}{2}+\frac{\pi}{4})\rvert)$
$e^{ax}\sin(bx), a^2+b^2 > 0$	$\frac{e^{ax}}{a^2+b^2}\,(a\sin(bx) - b\cos(bx))$
$e^{ax}\cos(bx), a^2+b^2 > 0$	$\frac{e^{ax}}{a^2+b^2}\,(a\cos(bx) + b\sin(bx))$
$\frac{1}{a^2+x^2}, a > 0$	$\frac{1}{a}\ \arctan(\frac{x}{a})$
$\frac{1}{a^2-x^2}, a > 0$	$\frac{1}{2a}\ \ln(\left\lvert\frac{a+x}{a-x}\right\rvert)$
$\frac{1}{\sqrt{a^2-x^2}}, a > 0$	$\arcsin(\frac{x}{a})$
$\frac{1}{\sqrt{a^2+x^2}}, a > 0$	$\ln(x+\sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}, a > 0$	$\ln(\left\lvert x+\sqrt{x^2-a^2}\right\rvert)$
$\sqrt{a^2-x^2}, a > 0$	$\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\ \arcsin(\frac{x}{a})$
$\sqrt{a^2+x^2}, a > 0$	$\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\ \ln(x+\sqrt{x^2+a^2})$
$\sqrt{x^2-a^2}, a > 0$	$\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\ \ln(\left\lvert x+\sqrt{x^2-a^2}\right\rvert)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\ln(\cosh(x))$

Remarks
All parameters are real numbers.
The constants of integration have been omitted.

Taylor Polynomials

Function	Taylor polynomial plus O-term
e^x	$1 + x + \frac{1}{2}x^2 + \cdots + \frac{1}{n!}x^n + O(x^{n+1})$
$\cos(x)$	$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots + \frac{(-1)^n}{(2n)!}x^{2n} + O(x^{2n+1})$
$\sin(x)$	$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + O(x^{2n+2})$
$\frac{1}{1+x}$	$1 - x + x^2 + \cdots + (-1)^n x^n + O(x^{n+1})$
$\ln(1+x)$	$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots + \frac{(-1)^n}{n+1}x^{n+1} + O(x^{n+2})$
$\frac{1}{1+x^2}$	$1 - x^2 + x^4 + \cdots + (-1)^n x^{2n} + O(x^{2n+1})$
$\arctan(x)$	$x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots + \frac{(-1)^n}{(2n+1)}x^{2n+1} + O(x^{2n+2})$
$(1+x)^\alpha$	$1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \cdots + \binom{\alpha}{n}x^n + O(x^{n+1})$

- All Taylor polynomials are polynomials around the point 0.
- The binomial coefficients are defined by

$$\binom{\alpha}{k} = \frac{\alpha \cdot (\alpha-1) \cdot (\alpha-2) \cdots (\alpha-(k-1))}{1 \cdot 2 \cdot 3 \cdots k}, \; k = 1, 2, 3, \ldots$$

$$\binom{\alpha}{0} = 1$$