EINDHOVEN UNIVERSITY OF TECHNOLOGY
Department of Mathematics and Computer Science

Examination Basic Mathematics, 2DL03, Wednesday 13 April 2011, 9.00–12.00 uur.

Write clearly the program (Pre-master program or TU/e-minor) you are following on the first page of your work.
The exam consists of 12 problems.
The answers and the computations should be written out well-formulated and well-organized.
It is not allowed to use a laptop, graphical or programmable calculator, chart with formulas, a book or other written material.
You may use a simple calculator simply and solely to check your answers.

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1. The function f is defined by $f(x) = x^2 + \dfrac{1}{x^2}$.

   Locate and classify the extreme values of the function f.
   Find the domain $D(f)$ and the range $R(f)$ of the function f.
   Sketch the graph of f.

2. Differentiate the expression $\arctan(\sqrt{x^2-1})$ with respect to x and simplify the result.
   Notation: $\arctan = \tan^{-1}$.

3. Determine the limit $\lim\limits_{x\to 1} \dfrac{|x^2-2|-x}{x-1}$.

4. Find all x which satisfy the inequality $x > \dfrac{x}{x+1}$.

5. Find all solutions x of the equation $\cos(x) = \cos^2(x) - \sin^2(x)$.

6. The function f is defined by $f(x) = \begin{cases} 1-\mathrm{e}^{-x} & , \quad x < 0\,, \\ x^3 & , \quad x \geq 0\,. \end{cases}$

   (a) Show that the function f is one-to-one.

   (b) Find the inverse f^{-1} of the function f.

7. (a) Find all exact x that satisfy $\sin(\arctan(x)) = \dfrac{1}{2}$.

   (b) Simplify the expression $\sin(\arctan(x))$.

   Notation: $\arctan = \tan^{-1}$.

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8. Consider the function f which is defined by $f(x) = \dfrac{1}{\sqrt{x}}$ for all $x > 0$.

   (a) Find the linearization of the function f about the point $a = 4$.

   (b) Find an approximation for $\dfrac{1}{\sqrt{3.92}}$ using the linearization from part (a).

   (c) Is the approximation from part (b) greater than $\dfrac{1}{\sqrt{3.92}}$?

9. Consider the functions f and g which are defined by $f(x) = \ln(1 + x)$ and by $g(x) = \ln(1 + x^2)$.

   (a) Determine the Taylor polynomial for f of the order 4 about $a = 0$.

   (b) Determine the Taylor polynomial for g of the order 4 about $a = 0$.

10. Consider the integral $I_0 = \displaystyle\int_0^{\pi/4} \frac{\tan(x)}{(\tan^2(x) + 3)^2} \cdot \frac{1}{\cos^2(x)}\,\mathrm{d}x$.

    (a) Transform the integral I_0 with the aid of the substitution $u = \tan^2(x) + 3$.

    (b) Compute the integral I_0.

11. Compute the integral $\displaystyle\int x^2 \ln^2(x)\,\mathrm{d}x$.

12. Show that $\displaystyle\int_0^1 \sqrt{9x^2 + x^8}\,\mathrm{d}x > \frac{3}{2}$ is true without computing the integral itself.

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The division of the points over the problems is as follows:

Problem	1:	3	points	Problem	6b:	2	points	Problem	9a:	2	points
Problem	2:	3	points	Problem	7a:	2	points		9b:	2	points
Problem	3:	3	points		7b:	2	points	Problem	10a:	2	points
Problem	4:	3	points	Problem	8a:	2	points	Problem	10b:	1	point
Problem	5:	3	points		8b:	1	point	Problem	11:	3	points
Problem	6a:	2	points		8c:	1	point	Problem	12:	3	points

The mark for the examination is obtained by dividing the total of scored points by 4 and rounding off to an integer.
