

Examination Basic Mathematics, 2DL03, Wednesday 27 June 2012, 9.00–12.00.

Write clearly the program (Pre-master program or TU/e-minor) you are following on the first page of your work.

The exam consists of 13 problems.

The answers and the computations should be written out well-formulated and well-organized.

It is not allowed to use a laptop, graphical or programmable calculator, chart with formulas, a book or other written material.

You may use a simple calculator simply and solely to check your answers.

1. Solve the inequality $\frac{2x}{x^3 - 2x} \leq 1$.
2. Consider the function f with $f(x) = 3 + \sqrt{2x - 1}$.
 - (a) Show that the function f is one-to-one.
 - (b) Find the domain $D(f^{-1})$ and the range $R(f^{-1})$ of the inverse function.
 - (c) Determine the inverse function $f^{-1}(x)$.
3. Compute the Taylor polynomial of the third order about $a = 1$ of the function $g(x) = \frac{1}{\sqrt{x}}$.
4. Determine the equation of the tangent line at the point $P : (-2, -1)$ to the curve \mathcal{C} , given by the equation $x^2 + 2y^4 = 3xy$.
5. Show the identity $\frac{1 - \tan^2(2x)}{\tan^2(2x) + 1} = \cos(4x)$.
6. Let $\varphi = \arctan\left(\frac{1}{2}\right)$.
Compute $\cos(\varphi)$ and $\sin(\varphi)$.
Remark about expression: $\arctan = \tan^{-1}$
7. Show that $\frac{\sqrt[3]{8+x} - 2}{x} \leq \frac{1}{3}$ for all x in \mathbb{R} with $x > 0$.
Hint: use the Mean-Value Theorem.
8. Consider the function f with $f(x) = \ln\left(\frac{x-2}{x^2}\right)$.
Compute the derivative $f'(x)$ and simplify it.

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9. Consider the function f with $f(x) = \tan(x)$.
- Determine the linearization of f about $a = \frac{\pi}{4}$.
 - Find an approximation for $f(\frac{\pi}{4} + \frac{1}{100})$ using the linearization of part (a).
 - Is the approximation of part (b) smaller than $f(\frac{\pi}{4} + \frac{1}{100})$?
10. Compute the limit $\lim_{x \rightarrow 1} \frac{4x^2 - x}{x - 1} - \frac{x^2 + x - 2}{(x - 1)^2}$.
11. Compute the integral $\int_1^4 \frac{\sqrt{x} + 1}{\sqrt{x}} e^{\sqrt{x}} dx$.
12. Consider the function F with $F(x) = \int_0^{x^2} e^{t^2} (1 - t) dt$.
- Find $F'(x)$.
Hint: do not compute the integral.
 - Find all x for which it is true that $F'(x) < 0$.
13. Compute the integral $\int \frac{x^2}{\sqrt{1 - x^2}} dx$.

The division of the points over the problems is as follows:

Problem 1:	2 points	Problem 5:	3 points	Problem 9c:	2 points
Problem 2a:	1 point	Problem 6:	3 points	Problem 10:	3 points
Problem 2b:	2 points	Problem 7:	3 points	Problem 11:	3 points
Problem 2c:	2 points	Problem 8:	2 points	Problem 12a:	2 points
Problem 3:	2 points	Problem 9a:	1 point	Problem 12b:	1 point
Problem 4:	3 points	Problem 9b:	2 points	Problem 13:	3 points

The mark for the examination is obtained by dividing the total of scored points by 4 and rounding off to an integer.
