Basiswiskunde, 2XL03,

dinsdag 6 november 2012

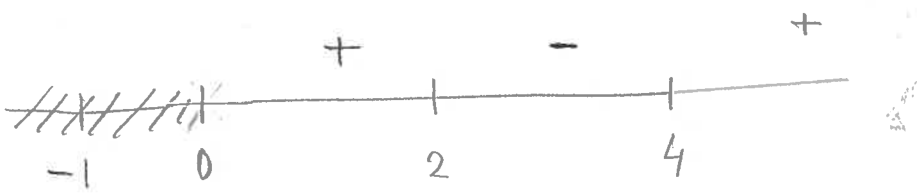
1)

$$\frac{x}{x-2} - \frac{\sqrt{x}\,(x-2)}{(x-2)} > 0$$

$$\frac{\sqrt{x}\left(\sqrt{x} - x + 2\right)}{x-2} > 0$$

$$\frac{\sqrt{x}\left(x - \sqrt{x} - 2\right)}{x-2} < 0$$

$$\frac{\sqrt{x}\left(\sqrt{x} - 2\right)\left(\sqrt{x} + 1\right)}{x-2} < 0$$



$$2 < x < 4$$

Teken $\dfrac{x}{x-2}$:

Opl: $2 < x < 4$

3) 2) volgt

$f(x) = \tan(x)$; $f\left(\frac{\pi}{4}\right) = 1$

$f'(x) = \cos^{-2}(x)$; $f'\left(\frac{\pi}{4}\right) = 2$

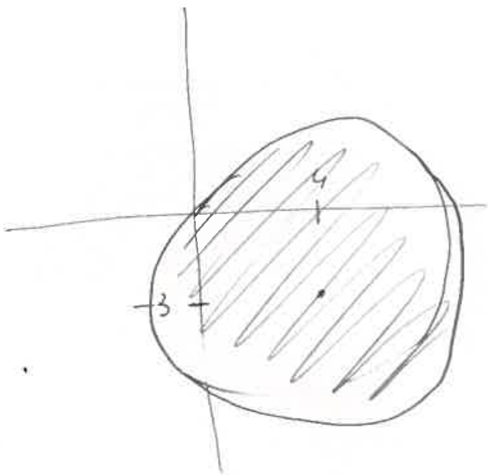
$f''(x) = +2\cos^{-3}(x)\sin(x)$; $f''\left(\frac{\pi}{4}\right) = +4$

2 a) $x^2 + y^2 \leq 8x - 6y$

$x^2 - 8x + 16 + y^2 + 6y + 9 \leq 25$

$(x-4)^2 + (y+3)^2 \leq 25$

Cirkelschijf met middelpunt $(4,-3)$ en straal 5. De rand doet mee.



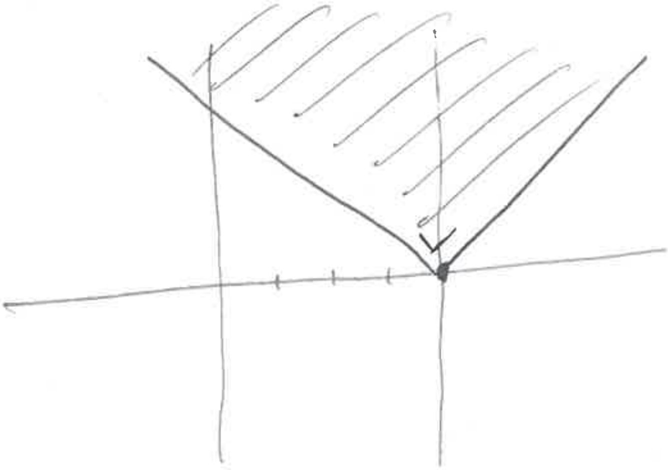
b) $y \geq |x-4|$

$x \geq 4 \qquad y \geq x - 4$

$x \leq 4 \qquad y \geq -x + 4$

Gebied wordt begrensd door twee lijnen. Grenzen doen mee.

Hoekpunt is $(4,0)$.

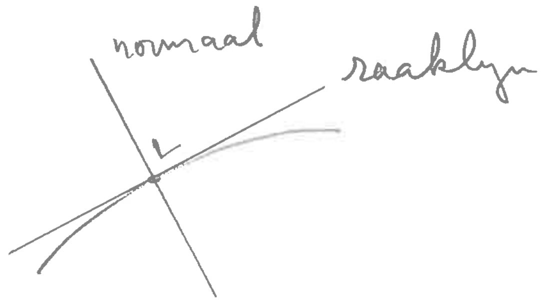


$$f'''(x) = +6\cos^{-4}(x)\sin^2(x) + 2\cos^{-2}(x);\quad f'''\left(\tfrac{\pi}{4}\right) = 16$$

$$P_3(x) = 1 + 2\left(x-\tfrac{\pi}{4}\right) + \frac{4}{2}\left(x-\tfrac{\pi}{4}\right)^2 + \frac{16}{6}\left(x-\tfrac{\pi}{4}\right)^3$$

$$= 1 + 2\left(x-\tfrac{\pi}{4}\right) + 2\left(x-\tfrac{\pi}{4}\right)^2 + \frac{8}{3}\left(x-\tfrac{\pi}{4}\right)^3$$

4)



Vgl $x^2 - y\ln(2-x) + y^2 = 5$

Diff naar x:

$$2x - y'\ln(2-x) + \frac{y}{2-x} + 2yy' = 0$$

$x = 1$ invullen

$$2 - y'(1)\cdot 0 + y(1) + 2y(1)\,y'(1) = 0$$

$$2 - 2 - 4y'(1) = 0$$

$$y'(1) = 0$$

Raaklijn: $y + 2 = 0(x-0)$

$$y = -2$$

Normaallijn moet door punt $(0, -2)$ gaan

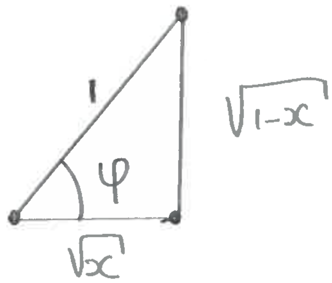
Raaklijn is horizontaal, normaallijn is verticaal

Vgl normaallijn $x = 0$

5)

$$\varphi = \arccos(\sqrt{x}) \quad \text{met} \quad 0 < x < 1$$

$$\cos(\varphi) = \sqrt{x} \quad \text{met} \quad 0 < x < 1$$



$$\sin(\varphi) = \sqrt{1-x}$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\cos^2(x) = \frac{1}{2}\cos(2x) + \frac{1}{2}$$

$$\cos^2\left(\frac{\varphi}{2}\right) = \frac{1}{2}\cos(\varphi) + \frac{1}{2}$$

$$\cos^2\left(\frac{\varphi}{2}\right) = \frac{1}{2}\sqrt{1-x} + \frac{1}{2}$$

$$\cos\left(\frac{\varphi}{2}\right) = \sqrt{\frac{1}{2}\sqrt{1-x} + \frac{1}{2}}$$

6)

$$\sqrt{\frac{1-\cos(2x)}{1+\cos(2x)}} = \sqrt{\frac{\cancel{2}\sin^2(x)}{\cancel{2}\cos^2(x)}} = \sqrt{\tan^2(x)} = \tan(x)$$

$$\left(0 \leq x < \frac{\pi}{2}\right)$$

7)

$$f(x) = \ln(2-x) \; ; \; f'(x) = -\frac{1}{2-x}$$

$$\ln(2-x) - \ln(2) = f(x) - f(0)$$

$$= (x-0) \cdot \frac{-1}{2-c}$$

met $x < c < 0$

$$\frac{1}{2-c} \leq \frac{1}{2} \quad \text{want} \quad c<0$$

Omdat $-x > 0$ vinden we

$$\frac{-x}{2-c} \leq -\frac{x}{2}$$

Dus

$$\ln(2-x) - \ln(2) \leq -\frac{x}{2}$$

8)

$$\lim_{x\to 0} \frac{\ln(1+2x)}{e^{3x}-1} =$$

$$\left[\begin{array}{lll} \ln(1+x) = x + O(x^2), & x \to 0 & (1) \\ e^{x} = 1 + x + O(x^2), & x \to 0 & (2) \end{array}\right.$$

[Verwang in (1) x door $2x$
in (2) x door $3x$

$$= \lim_{x\to 0} \frac{2x + O(x^2)}{3x + O(x^2)} =$$

$$= \lim_{x\to 0} \frac{2 + O(x)}{3 + O(x)} = \frac{2+0}{3+0} = \frac{2}{3}$$

9) $\displaystyle\int_0^1 \frac{x^5}{x^3+1}\,dx =$

$$\left[\begin{array}{l} u = x^3+1 \\ du = 3x^2\,dx \\ x=0,\ u=1\ ;\ x=1,\ u=2 \end{array}\right.$$

$$= \frac{1}{3}\int_1^2 \frac{u-1}{u}\,du =$$

$$= \frac{1}{3}\int_1^2 \left(1-\frac{1}{u}\right) du =$$

$$= \frac{1}{3}\Big[u-\ln(u)\Big]_1^2 =$$

$$= \frac{1}{3}\Big(2-\ln(2)-1-0\Big)$$

$$= \frac{1}{3}\Big(1-\ln(2)\Big)$$

10) $\displaystyle\int \sqrt{x}\, e^{\sqrt{x}}\,dx =$

$$\left[\begin{array}{l} u = \sqrt{x} \\ du = \dfrac{1}{2\sqrt{x}}\,dx \\ x = u^2\ ;\ dx = 2u\,du \end{array}\right.$$

$$= 2\int \underbrace{u^2}_{f(u)}\underbrace{e^{u}}_{g'(u)}\,du =$$

$$= 2u^2e^u - 4\int u e^u\,du =$$

$$= 2u^2e^u - 4ue^u + 4\int e^u du =$$

$$= 2u^2e^u - 4ue^u + 4e^u + c, \ c \in \mathbb{R}$$

$$= 2xe^{\sqrt{x}} - 4\sqrt{x}e^{\sqrt{x}} + 4e^{\sqrt{x}} + c, \ c \in \mathbb{R}$$

11) a)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sin(2x)} dx =$$

$$\left[\begin{array}{l} u = 2x \\ du = 2\,dx \\ \\ x = \frac{\pi}{4},\ u = \frac{\pi}{2};\ x = \frac{\pi}{3};\ u = \frac{2}{3}\pi \end{array} \right.$$

$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{2}{3}\pi} \frac{1}{\sin(u)} du =$$

$$= \frac{1}{2} \left[\ln\left(\left|\tan\left(\frac{u}{2}\right)\right|\right) \right]_{\frac{\pi}{2}}^{\frac{2}{3}\pi}$$

$$= \frac{1}{2}\left(\ln\left(\tan\left(\frac{\pi}{3}\right)\right) - \ln\left(\tan\left(\frac{\pi}{4}\right)\right)\right)$$

$$= \frac{1}{2}\left(\ln(\sqrt{3}) - \ln(1)\right)$$

$$= \frac{1}{4}\ln(3)$$

b) Breedte interval $\frac{\pi}{3} - \frac{\pi}{4} = \frac{4}{12}\pi - \frac{3}{12}\pi =$

$$= \frac{\pi}{12}.$$

Gemiddelde waarde:

$$\frac{12}{\pi} \cdot \frac{1}{4} \ln(3) = \frac{3}{\pi} \ln(3)$$

12) $F(x) = \int_{x^2}^{1} \frac{t^2}{1+e^t}\, dt$

$$= - \int_{1}^{x^2} \frac{t^2}{1+e^t}\, dt.$$

$$F'(x) = - \frac{x^4}{1+e^{x^2}} \cdot 2x = - \frac{2x^5}{1+e^{x^2}}.$$