Tentamen Basiswiskunde, 2XL03,
Maandag 15 april 2013

Opgave 1

In[1]:= $\text{Reduce}\left[\frac{x}{2-x} \geq x^2, x\right]$

Out[1]= $0 \leq x < 2$

In[2]:= $\text{Reduce}\left[x \geq x^2 (2-x), x\right]$

Out[2]= $x \geq 0$

In[3]:= $\text{Reduce}\left[\frac{1}{2-x} \geq x, x\right]$

Out[3]= $x < 2$

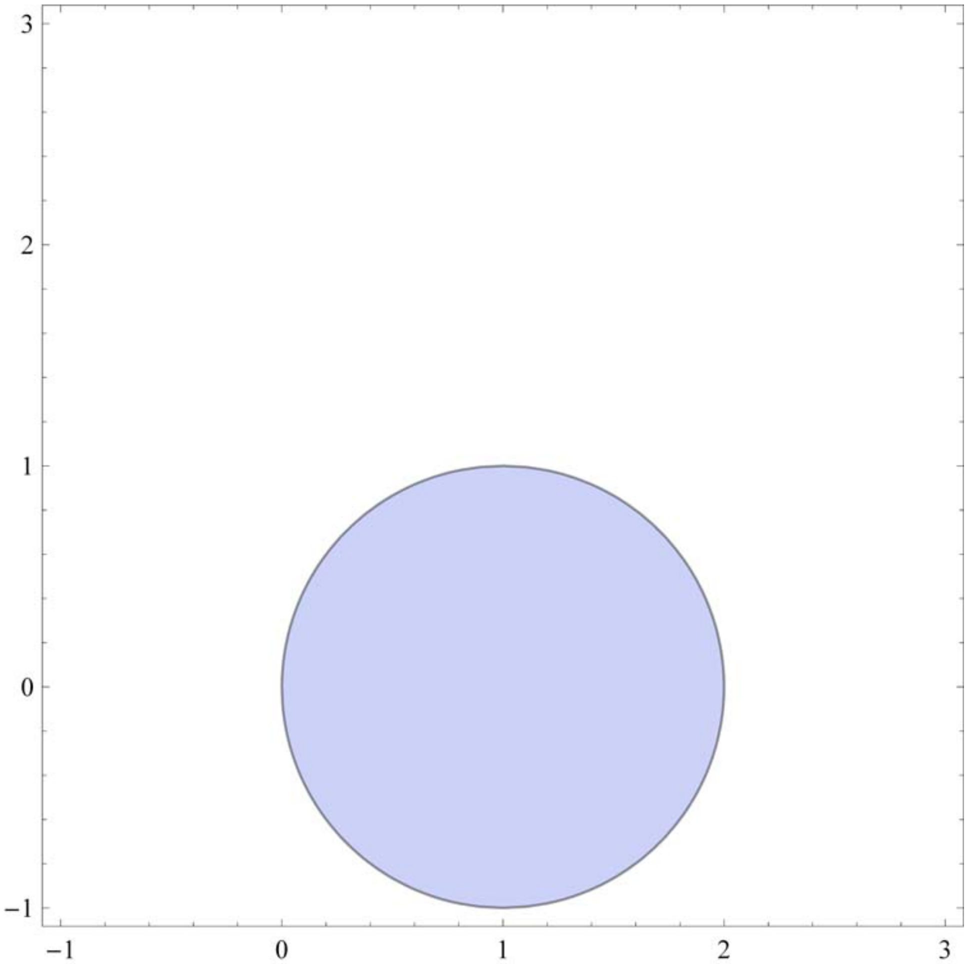
In[4]:= $\text{Reduce}[1 \geq x (2-x), x]$

Out[4]= $x \in \text{Reals}$

Opgave 2

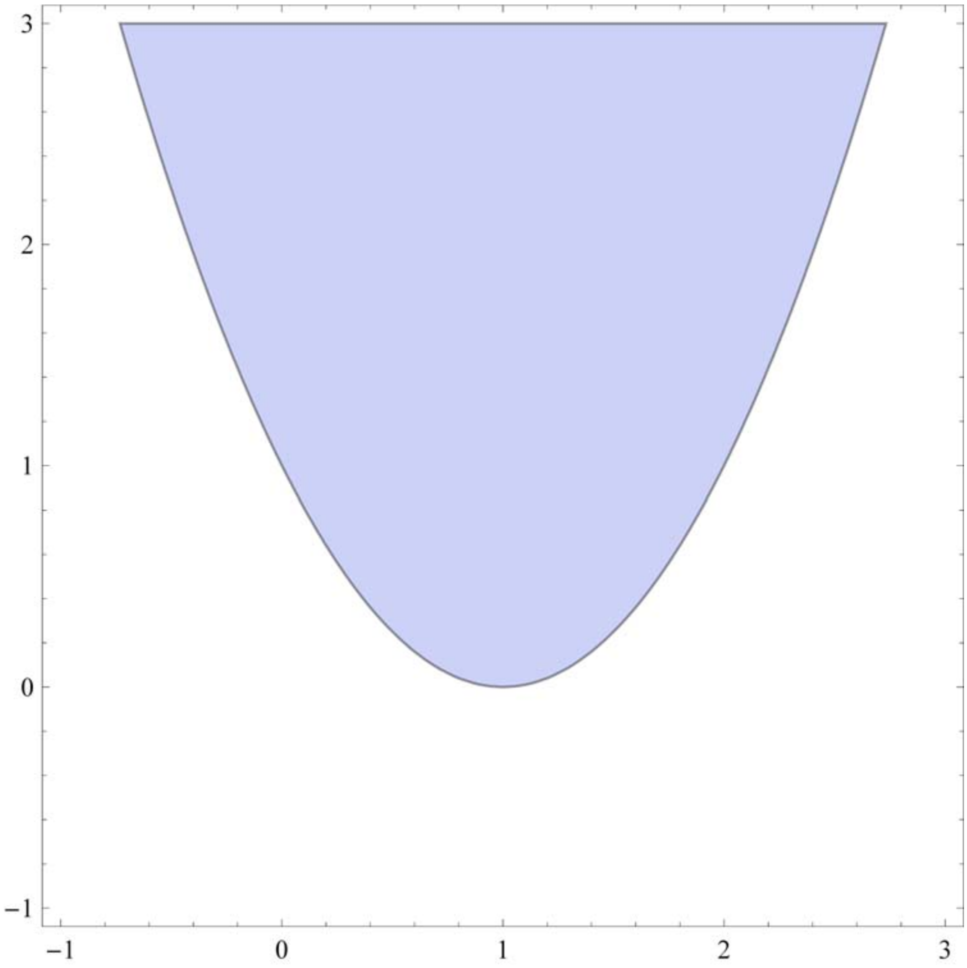
In[5]:= RegionPlot[$x^2 + y^2 \leq 2x$, {x, -1, 3}, {y, -1, 3}]

Out[5]=



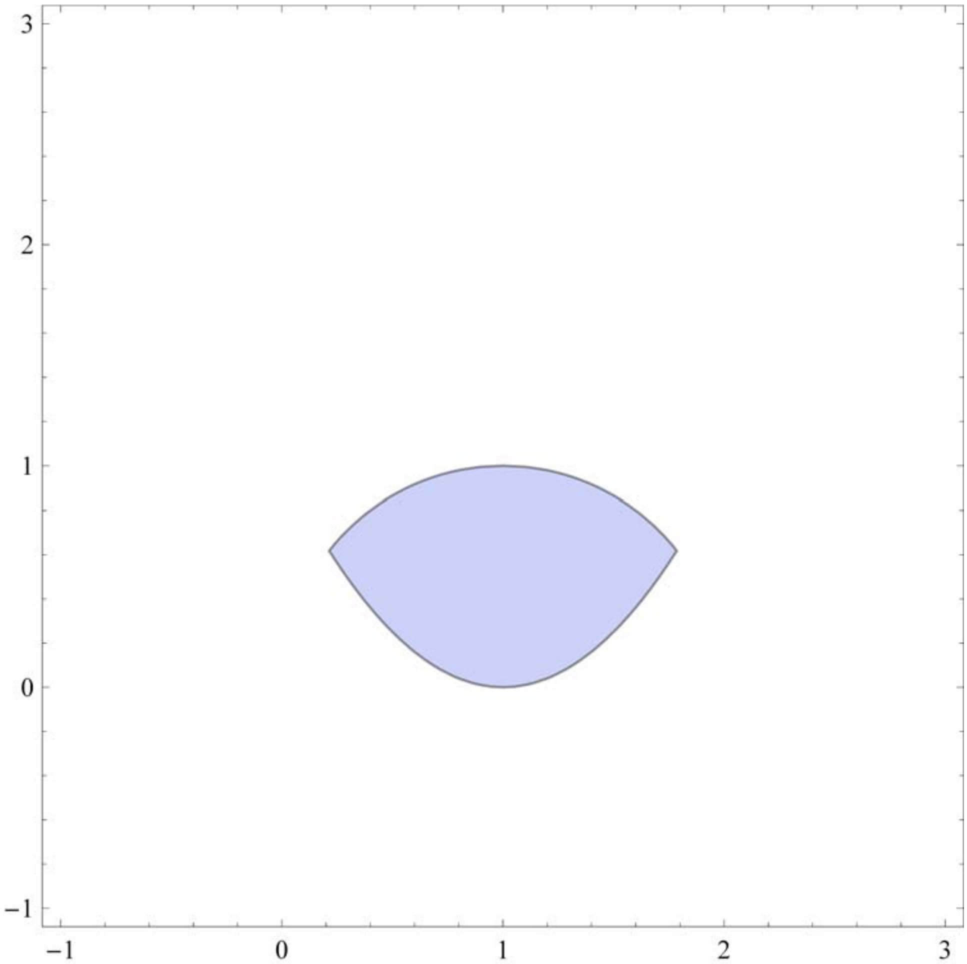
In[6]:= `RegionPlot[y ≥ (x - 1)^2, {x, -1, 3}, {y, -1, 3}]`

Out[6]=



In[7]:=
```
RegionPlot[x^2 + y^2 ≤ 2 x && y ≥ (x - 1)^2, {x, -1, 3},
  {y, -1, 3}]
```

Out[7]=



Opgave 3

In[8]:=
```
ClearAll[f, p2, x];

f[x_] := Tan[x]

p2[x_] = Normal[Series[f[x], {x, π/6, 2}]]
```

Out[10]=
$$\frac{1}{\sqrt{3}} + \frac{4}{3}\left(-\frac{\pi}{6} + x\right) + \frac{4\left(-\frac{\pi}{6} + x\right)^2}{3\sqrt{3}}$$

In[11]:= f''[x]

Out[11]= $2\,\text{Sec}[x]^2\,\text{Tan}[x]$

In[12]:= $2\,\frac{\text{Sin}[x]}{\text{Cos}[x]^3}$

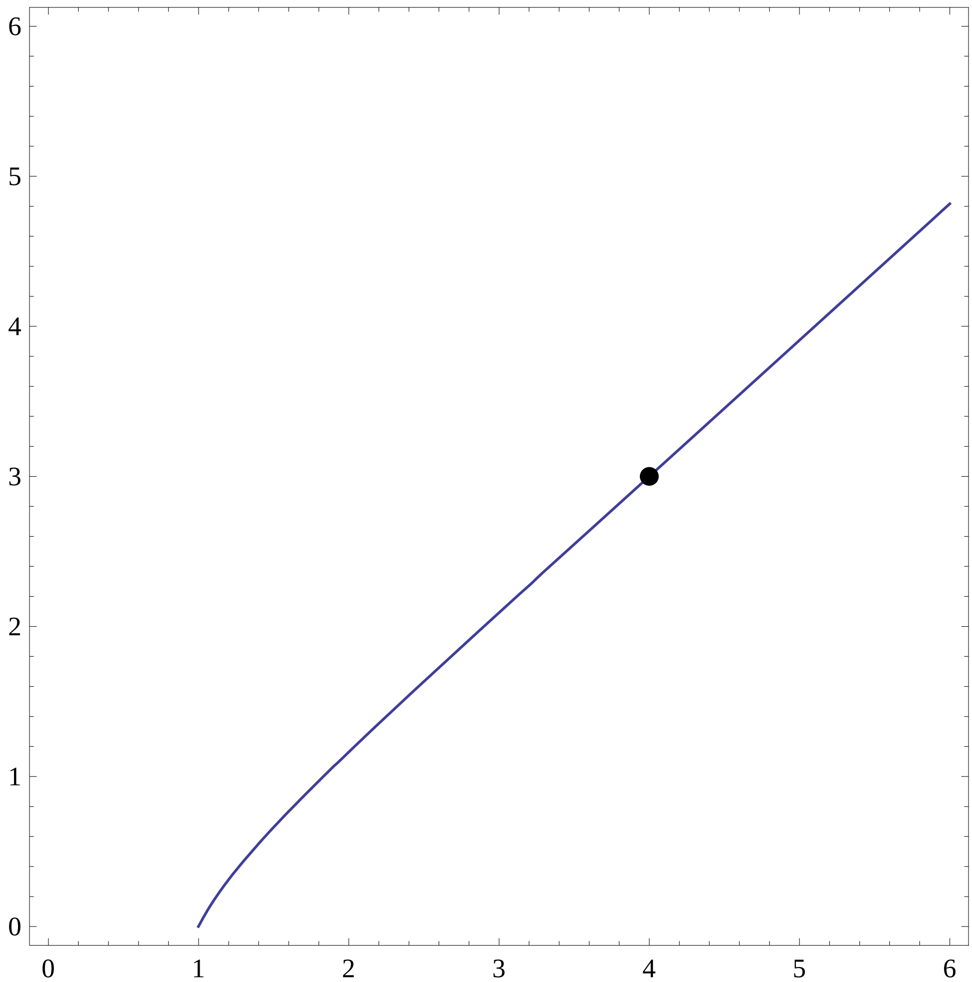
Out[12]= $2\,\text{Sec}[x]^2\,\text{Tan}[x]$

Opgave 4

In[13]:= eq1 = $x^2 - y\sqrt{x} - y^2 == 1$;

In[14]:=
```
pl1 = ContourPlot[Evaluate[eq1], {x, 0, 6}, {y, 0, 6},
  Epilog → {{PointSize[0.02], Point[{4, 3}]}}]
```

Out[14]=



In[15]:=
```
eq2 = eq1 /. y → y[x]
```

Out[15]= $x^2 - \sqrt{x}\, y[x] - y[x]^2 == 1$

In[16]:=
```
eq3 = D[eq2, x]
```

Out[16]= $2x - \frac{y[x]}{2\sqrt{x}} - \sqrt{x}\, y'[x] - 2\, y[x]\, y'[x] == 0$

In[17]:=
```
(eq2 /. x → 4) /. y[4] → 3
```

Out[17]= True

In[18]:=
```
eq4 = (eq3 /. x → 4) /. y[4] → 3
```

Out[18]= $\frac{29}{4} - 8\, y'[4] == 0$

In[19]:=
```
Solve[eq4, y'[4]]
```

Out[19]= $\left\{\left\{y'[4] \to \frac{29}{32}\right\}\right\}$

In[20]:=
```
Simplify[y - 3 == 29/32 (x - 4)]
```

Out[20]= $29\,x == 20 + 32\,y$

In[21]:=
```
pl2 = ContourPlot[y - 3 == 29/32 (x - 4), {x, 0, 6},
    {y, 0, 6},
    Epilog → {{PointSize[0.02], Point[{4, 3}]}}];
```

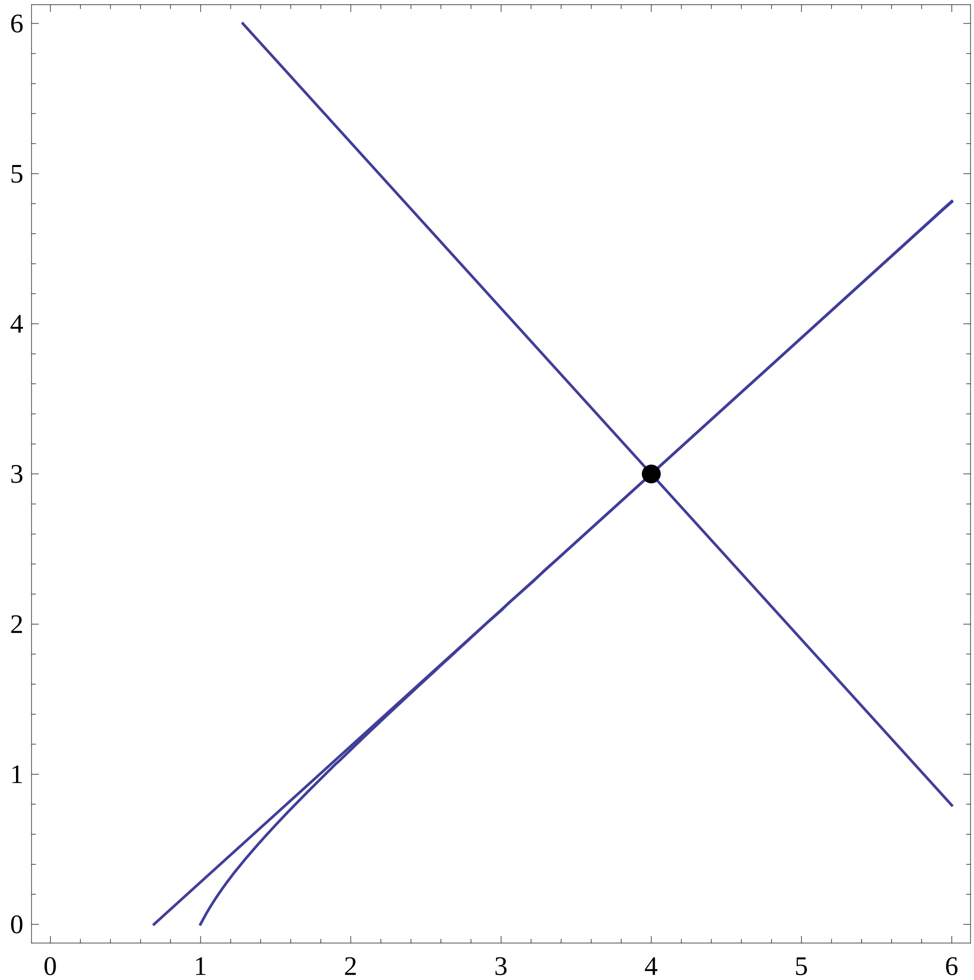
In[22]:=
```
pl3 = ContourPlot[y - 3 == -32/29 (x - 4), {x, 0, 6},
    {y, 0, 6},
    Epilog → {{PointSize[0.02], Point[{4, 3}]}}];
```

In[23]:= **Simplify**$\left[y - 3 == -\frac{32}{29}\,(x - 4)\right]$

Out[23]= $32\,x + 29\,y == 215$

In[24]:= **Show[pl1, pl2, pl3]**

Out[24]=



Opgave 5

In[25]:= φ = **ArcTan**$\left[\frac{1}{x}\right]$

Out[25]= ArcTan$\left[\frac{1}{x}\right]$

In[26]:= `FullSimplify[Sin[φ], Assumptions → x > 0]`

Out[26]= $$\frac{1}{\sqrt{1+x^2}}$$

In[27]:= `FullSimplify[Cos[φ], Assumptions → {x > 0}]`

Out[27]= $$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

In[28]:= `FullSimplify[2 Sin[φ] Cos[φ]]`

Out[28]= $$\frac{2\,x}{1+x^2}$$

Opgave 6

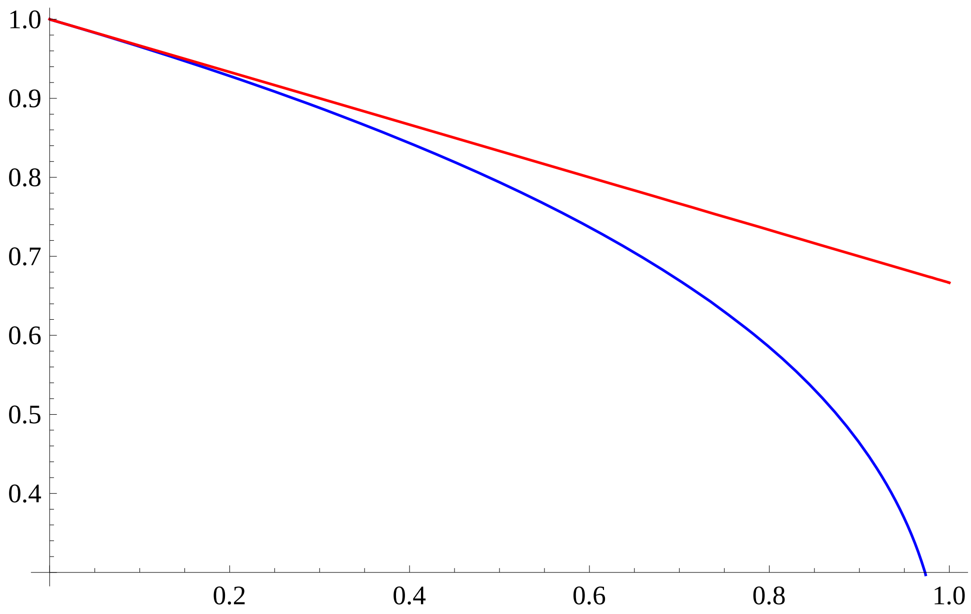
In[29]:= $$\text{FullSimplify}\left[\sqrt{\frac{1-\text{Cos}[2\,x]}{1+\text{Cos}[2\,x]}}\right]$$

Out[29]= $$\sqrt{\text{Tan}[x]^2}$$

Opgave 7

In[30]:=
```
Plot[{∛(1 - x), -x/3 + 1}, {x, 0, 1},
  PlotStyle → {Blue, Red}]
```

Out[30]=



Laat $f(x) = \sqrt[3]{1-x}$. Dan $f'(x) = -\frac{1}{3}(1-x)^{-2/3}$.

Dus $f(x) - f(0) = \sqrt[3]{1-x} - 1 = -\frac{x}{3} \frac{1}{\sqrt[3]{(1-c)^2}}$ met $0 < c < x < 1$

Omdat $\sqrt[3]{(1-c)^2} < 1$, is $\frac{1}{\sqrt[3]{(1-c)^2}}$>1 en dus $-\frac{x}{3} \frac{1}{\sqrt[3]{(1-c)^2}} < -\frac{x}{3}$ en

dus

$\sqrt[3]{1-x} - 1 < -\frac{x}{3}$ ofwel $\sqrt[3]{1-x} < -\frac{x}{3} + 1$

Opgave 8

In[31]:=
```
Limit[(ⅇ^x - 1)/Log[1 + x], x → 0]
```

Out[31]= 1

Opgave 9

In[32]:=
```
ClearAll[f, l, x]

f[x_] := Cos[x]^2

l[x_] = Normal[Series[f[x], {x, π/3, 1}]]
```

Out[34]= $\frac{1}{4}-\frac{1}{2}\sqrt{3}\left(-\frac{\pi}{3}+x\right)$

Opgave 10

In[35]:=
```
int1 = Integrate[Sqrt[2 - 2 Cos[2 t]], t]
```

Out[35]= $-2\,\mathrm{Cot}[t]\sqrt{\mathrm{Sin}[t]^2}$

In[36]:=
```
PowerExpand[int1]
```

Out[36]= $-2\,\mathrm{Cos}[t]$

In[37]:=
```
int2 = Integrate[Sqrt[2 - 2 Cos[2 t]], {t, 0, π/3}]
```

Out[37]= 1

Opgave 11

In[38]:= `int1 = Integrate[Sqrt[x] E^(x Sqrt[x]), x]`

Out[38]= $\frac{2\, e^{x^{3/2}}}{3}$

In[39]:= `int2 = Integrate[Sqrt[x] E^(x Sqrt[x]), {x, 0, 1}]`

Out[39]= $\frac{2}{3}\,(-1 + e)$

Opgave 12

In[40]:= `Integrate[E^(t^2)/(1 + t^2), t]`

Out[40]= $\int \frac{e^{t^2}}{1 + t^2}\, dt$

In[41]:= `expr = Integrate[E^(t^2)/(1 + t^2), {t, 1, x^2}]`

Out[41]= $\int_1^{x^2} \frac{e^{t^2}}{1 + t^2}\, dt$

In[42]:= `D[expr, x]`

Out[42]= $\frac{2\, e^{x^4}\, x}{1 + x^4}$