

**Examination Basic Mathematics, 2XL03, Monday, June 24, 2013, 14:00–17:00.**

---

Write clearly the program (Pre-master program or TU/e-minor) which you are following on the first page of your work.

The exam consists of 12 problems.

The answers and the computations should be written out well-formulated and well-organized.

It is not allowed to use a laptop, graphical or programmable calculator, written material. You may use a simple calculator to check your answers.

---

1. Find the solutions  $x$  of the inequality  $x^2 \leq x\sqrt{x+2}$ .
2. Sketch in the plane the set of points  $(x, y)$  that satisfy  $x + |y| \leq 4$ .
3. The curve  $C$  passes through the point  $(0, -1)$  and is implicitly given by the equation  $x \ln(1 + y^2) - y^2 = -1$ .  
Find the equation of the tangent to  $C$  at the point  $(0, -1)$ .
4. Consider the function  $f$  defined by  $f(x) = \frac{\sqrt{2}}{\sqrt{1+x}}$  for  $x > -1$ .
  - (a) Determine the linearisation of  $f$  about  $a = 1$ .
  - (b) Approximate  $\frac{\sqrt{2}}{\sqrt{2.1}}$  by using the linearisation from part (a).
  - (c) Is the approximation from (b) greater than  $\frac{\sqrt{2}}{\sqrt{2.1}}$ ?
5. Find the limit  $\lim_{x \rightarrow 0} \frac{\arctan(x) - x}{x^3}$ .  
Expression:  $\arctan(x) = \tan^{-1}(x)$
6. The function  $f$ , defined by  $f(x) = x + \sqrt{1+x^2}$  for all  $x$  in  $\mathbb{R}$ , is one-to-one.  
Find the inverse  $f^{-1}(x)$ .
7. Show that  $\frac{4 \cos^4(x)}{\sin^2(2x)} + 1 = \frac{1}{\sin^2(x)}$ .

see next page

8. Show that for all  $x \in \mathbb{R}$  with  $x > 0$  it is true that  $\arctan\left(\frac{x}{2} + 1\right) - \frac{\pi}{4} \leq \frac{x}{4}$  by using the Mean-Value Theorem.

Expression:  $\arctan(x) = \tan^{-1}(x)$

9. Consider the integral  $\int_0^3 \frac{x e^{\sqrt{1+x}}}{\sqrt{1+x}} dx$ .

- (a) Rewrite the integral by using the substitution  $u = \sqrt{1+x}$ .  
 (b) Find the value of the integral.

10. Evaluate  $\int \frac{1+x}{\sqrt{1+x^2}} dx$ .

Hint: use the table.

11. Show that  $\int_0^1 \frac{e^x}{5+x^4} dx \leq \frac{e-1}{5}$  without computing the integral.

12. Simplify the expression  $\frac{d}{dx} \left( \int_{\sin(x)}^1 \frac{t}{1+e^t} dt \right)$  without computing the integral.
- 

The division of the points over the problems is as follows:

Problem 1: 3 points	Problem 4c: 2 points	Problem 9a: 2 points
Problem 2: 3 points	Problem 5: 3 points	9b: 2 points
Problem 3: 3 points	Problem 6: 3 points	Problem 10 : 3 points
Problem 4a: 2 points	Problem 7: 3 points	Problem 11 : 3 points
4b: 2 points	Problem 8: 3 points	Problem 12 : 3 points

The mark for this exam (2XL03) is obtained by dividing the total of scored points by 4 and by rounding off to an integer.

The mark for the subject Basic mathematics (2DL03) is based on 90 % of the result of this exam (2XL03) and on 10 % of the result of the entrance test (2DA00). If you did not take the entrance test, then the mark for the subject Basic Mathematics (2DL03) is equal to the mark of this exam.

The lecturer will contact you about taking an entrance test if you should pass the examination with an entrance test.

---