# Homework Advanced Calculus (2DBN10)
# Set 3

**1.** (**Conic sections**)[1]

Let $K$ be the double cone surface in $\mathbb{R}^3$ given by

$$z^2 = (x-1)^2 + y^2.$$

Let $m > 0$ and let $V = V(m)$ be the plane with equation $z = mx$.

Let $L = L(m) = K \cap V(m)$ be the curve of intersection of $K$ and $V(m)$.

**a)** For which values of $m$ is $L$ an ellipse / parabola / hyperbola ? Give reasons for your answer. — 3 pt

(Hint: First, find an equation for the projection of $L$ to the $x, y$–plane.)

**b)** In the case where $L$ is an ellipse, find the center and the half axes. — 1 pt

(Caution: This is about the ellipse itself, not its projection.)

**c)** In the case where $L$ is a parabola, give the coordinates of its vertex. — 1 pt

**2.** Find equations of the form $p(x, y, z) = 0$ with $p$ a polynomial in three variables for the following sets of points in $\mathbb{R}^3$:

**a)** the union of the line through the points $(0, 1, 0)$ and $(1, 0, 0)$ and the $(x, z)$-plane, — 1 pt

**b)** the surface given by the parameterization — 1 pt

$$x(u, v) = u, \; y(u, v) = u\cos v, \; z(u, v) = 2u\sin v, \quad u \in \mathbb{R}, \; v \in [0, 2\pi),$$

**c)** the surface obtained by rotating the parabola given by $y = z^2 + 1$, $x = 0$ around the $z$-axis. — 1 pt

In b), verify your result by calculating $p(x(u, v), y(u, v), z(u, v))$.

**3.** Find parameter representations for the following surfaces in $\mathbb{R}^3$:

**a)** the surface given by the equation — 1 pt

$$x^2 - 2y^2 - z^2 = 2$$

(Hint: Use trigonometric and hyperbolic functions.)

**b)** the surface obtained by rotating the parabola given by $y = z^2 + 1$, $x = 0$ around the $z$-axis. — 1 pt

Verify your results using the equations for the surfaces (for b), see 2c)).

**Exam type problem (not to be handed in):** (possibly as part of some other problem)

Find a parameterization for the ellipsoid in $\mathbb{R}^3$ with equation

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 + \left(\frac{z - z_0}{c}\right)^2 = 1, \qquad a, b, c > 0.$$

[1] This problem is similar but not identic to the problem on conic section in last year's homework.