

Exercises Advanced Calculus (2DBN10)

Lecture 3

1. Find the general solution for the following linear inhomogeneous ODE systems:

a)

$$\begin{aligned}y_1' &= -3y_1 + y_2 + 3 \cos(t), \\y_2' &= y_1 - 3y_2 - 2 \cos(t) - 3 \sin(t),\end{aligned}$$

b)

$$\begin{aligned}y_1' &= -3y_1 - 4y_2 + 5e^t, \\y_2' &= 5y_1 + 6y_2 - 6e^t.\end{aligned}$$

2. Find the general solutions for the following linear homogeneous ODEs:

a) $u''' - u'' + 9u' - 9u = 0$,

b) $u'''' + 3u'' - 4u = 0$.

3. Use the method of trial solutions to find particular solutions to the following inhomogeneous ODEs:

a) $u''' - u'' + 9u' - 9u = e^{-t}$,

b) $u'''' + 3u'' - 4u = \sin t$.

4. Let $p > 0$ and $q < 0$. Show: The fourth order homogeneous ODE

$$u'''' + pu'' + qu = 0$$

has

- solutions u with $u(t) \rightarrow \infty$ as $t \rightarrow \infty$,
- nonzero solutions u with $u(t) \rightarrow 0$ as $t \rightarrow \infty$, and
- nonzero periodic solutions.