Solutions to exercises Advanced Calculus (2DBN10) Lecture 4

(These solutions are deliberately sketchy in part. No rights can be derived from them.)

3. **a)** Partial fraction decomposition:

$$\frac{s-9}{s^2-9} = \frac{2}{s+3} - \frac{1}{s-3}$$

Inverse Laplace transform:

$$t \mapsto 2e^{-3t} - e^{3t}.$$

b) Partial fraction decomposition:

$$\frac{1}{s^2(s+1)} = \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}.$$

Inverse Laplace transform:

$$t \mapsto t - 1 + e^{-t}.$$

c) Partial fraction decomposition:

$$\frac{s^3+6s^2+14s}{(s+2)^4} = \frac{1}{s+2} + \frac{2}{(s+2)^3} - \frac{12}{(s+2)^4}$$

Inverse Laplace transform:

$$t \mapsto e^{-2t}(1 + t^2 - 2t^3).$$

d) Partial fraction decomposition:

$$\frac{8}{s^4+4s^2} = \frac{2}{s^2} - \frac{2}{4+s^2}$$

Inverse Laplace transform:

$$t \mapsto 2t - \sin(2t).$$

e) Inverse Laplace transform:

$$t \mapsto \begin{cases} \frac{1}{6}(t-1)^3 & \text{for } t \geq 1, \\ 0 & \text{for } t < 1. \end{cases}$$

f) Inverse Laplace transform:

$$t \mapsto \begin{cases} \frac{1}{2}\sin(2t) & \text{for } t \in [0, \pi], \\ 0 & \text{for } t > \pi. \end{cases}$$

4. **a)** Let $Y = \mathcal{L}[y]$, $R = \mathcal{L}[r]$. Then

$$(s^2 - 5s + 6)Y(s) - s + 7 = R(s) = 4\frac{1 - e^{-2(s-1)}}{s-1}.$$

Solve for $Y(s)$ and partial fraction decomposition:

$$Y(s) = \frac{2}{s-1} + \frac{1}{s-2} - \frac{2}{s-3} - e^2 e^{-2s}\left(\frac{2}{s-1} - \frac{4}{s-2} + \frac{2}{s-3}\right).$$

Inverse transform:

$$y(t) = \begin{cases} 2e^t + e^{2t} - 2e^{3t} & \text{for } t \in [0, 2], \\ (4e^{-2} + 1)e^{2t} - (2e^{-4} + 2)e^{3t} & \text{for } t > 2. \end{cases}$$

b) Let $Y = \mathcal{L}[y]$, $R = \mathcal{L}[r]$. Then

$$(s^2 + 9)Y(s) - 4 = R(s) = 8\frac{1 + e^{-\pi s}}{s^2 + 1}.$$

Solve for $Y(s)$ and partial fraction decomposition:

$$Y(s) = \frac{1}{s^2+1} + \frac{3}{s^2+9} + e^{-\pi s}\left(\frac{1}{s^2+1} - \frac{1}{s^2+9}\right).$$

Inverse transform:

$$y(t) = \begin{cases} \sin t + \sin(3t) & \text{for } t \in [0, \pi], \\ \frac{4}{3}\sin(3t) & \text{for } t > \pi. \end{cases}$$