TECHNISCHE UNIVERSITEIT EINDHOVEN
Department of Mathematics and Computer Science

Final test Advanced Calculus (2DBN10), Thursday November 9, 2017, 9:00-12:00 .

A table of Laplace transforms is part of the problem sheet. No other materials (books, notes, computers, calculators etc.) are allowed.

The answers to the problems have to be formulated and motivated clearly.

1. **a)** Find the general solution to the differential equation

$$y'''(t) + 5y''(t) + 9y'(t) + 5y(t) = e^{-t} + e^{t}.$$

(3 points)

b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function with $f(t) = 0$ for $t \geq 2017$. Show: For each solution u of the differential equation

$$u'''(t) + 5u''(t) + 9u'(t) + 5u(t) = f(t)$$

we have $\lim_{t\to\infty} u(t) = 0$. (2 points)

2. Let y be the solution of the initial value problem

$$y'''(t) - 3y''(t) + 3y'(t) - y(t) = 0, \qquad y(0) = 1, \quad y'(0) = y''(0) = 0.$$

a) Calculate the Laplace transform $s \mapsto Y(s)$ van y. (2 points)

b) Calculate the solution y. (3 points)

3. Let the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be given by

$$f(x, y) = \frac{x^2 - 1}{y^2 + 1}.$$

a) Sketch the level sets of f for the function values 1, 0, and $-\frac{1}{2}$. (2 points)

b) Give an equation for the tangent to the level curve through the point $(2, 1)$. (1 point)

c) Give an equation for the tangent plane to the graph of f in the point $(2, 1, 3/2)$. (1 point)

4. Give the second order Taylor polynomial around the point $(x_0, y_0, z_0) = (1, 0, \pi)$ for the function f given by

$$f(x, y, z) = \frac{\cos z}{x + y}.$$

(5 points)

please turn over

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5. Let the function f be given by

$$f(x,y) = \frac{1}{x} + \frac{4}{y} + \frac{9}{4-x-y}$$

Find the critical points of f and determine their type. (6 POINTS)

6. Find the maximal value of the function $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ given by

$$f(x,y,z) = xy^2z$$

on the ellipsoid in $\mathbb{R}^3$ given by

$$x^2 + 3y^2 + 2z^2 = 64.$$

(Hint: Argue first that you can restrict yourself to the case $x > 0$, $y > 0$, $z > 0$.) (5 POINTS)

7. Consider the system of equations

$$\begin{aligned} \sin(xt) + x + y &= 0, \\ \cos(yt) + x + 2y + t &= 2. \end{aligned}$$

a) Under which assumptions on the point (t_0, x_0, y_0) is this system solvable in the form $x = \phi(t)$, $y = \psi(t)$ near this point? (2 POINTS)

b) Verify that the point $(t_0, x_0, y_0) = (1, 0, 0)$ satisfies these assumptions, and calculate $\phi'(1)$ and $\psi'(1)$. (3 POINTS)

8. Let K be the body in $\mathbb{R}^3$ given by

$$K = \{(x,y,z) \,|\, x \geq y^2,\ x - y \leq 2,\ 0 \leq z \leq x\}.$$

Fing the volume of K. (5 POINTS)

Het cijfer wordt bepaald door het totaal der behaalde points te delen door 4 en af te ronden op een cijfer achter de komma.

Some standard Laplace transforms and calculation rules:

(schematically, $n \in \mathbb{N}_+$, $\omega \neq 0$, $a > 0$, $b \in \mathbb{R}$)

$F(s) = \mathcal{L}[f](s)$	$f(t)$
$sF(s) - f(0)$	$f'(t)$
$s^n F(s) - \sum_{k=0}^{n-1} s^k f^{(n-1-k)}(0)$	$f^{(n)}(t)$
$\dfrac{F(s)}{s}$	$\int_0^t f(\tau)\, d\tau$
$\frac{1}{a} F\left(\frac{s}{a}\right)$	$f(at)$
$F(s-b)$	$e^{bt} f(t)$
$e^{-as} F(s)$	$\begin{cases} f(t-a) & \text{if } t \geq a, \\ 0 & \text{if } t < a \end{cases}$
$\dfrac{1}{s^n}$	$\dfrac{t^{n-1}}{(n-1)!}$
$\dfrac{1}{s^2 + \omega^2}$	$\frac{1}{\omega} \sin(\omega t)$
$\dfrac{s}{s^2 + \omega^2}$	$\cos(\omega t)$
$\dfrac{1}{(s^2 + \omega^2)^2}$	$\frac{1}{2\omega^3} (\sin(\omega t) - \omega t \cos(\omega t))$
$\dfrac{s}{(s^2 + \omega^2)^2}$	$\frac{1}{2\omega} t \sin(\omega t)$