Typical errors in the final test Analysis 1 (2WA31), January 2014

We learn from failure, not from success!
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This document contains a number of frequently encountered errors in the final test of the course Analysis 1 (2WA31, January 2014). Try to find and correct them.

2. What goes wrong?

"For $k = 0, \dots n$ we have $\lim_{n\to\infty} \frac{1}{n+k} = 0$. So

$$\begin{aligned}\lim_{n\to\infty} a_n &= \lim_{n\to\infty}\left(\frac{1}{n} + \frac{1}{n+1} + \ldots + \frac{1}{2n}\right)\\ &= \lim_{n\to\infty}\frac{1}{n} + \lim_{n\to\infty}\frac{1}{n+1} + \ldots + \lim_{n\to\infty}\frac{1}{2n}\\ &= 0 + 0 + \ldots + 0 = 0.\text{ ''}\end{aligned}$$

3. What goes wrong? How to correct this?

"...Let v be an approximation point of the sequence (c_n). Then there is a subsequence (c_{n_k}) with $c_{n_k} \to v$ as $k \to \infty$. For the corresponding subsequences (a_{n_k}) and (b_{n_k}) we have: $a_{n_k} \to 1$ or $a_{n_k} \to 2$ as $k \to \infty$, and also $b_{n_k} \to 2$ or $b_{n_k} \to 4$ als $k \to \infty$. According to the calculation rules for limits we get $v \in \{2, 4, 8\}$. So $V \subseteq \{2, 4, 8\}$."

4.a) What goes wrong? How to do this correctly?

"According to a standard Taylor expansion we have

$$1 - \cos(x) = \frac{x^2}{2} + O(x^4).$$

The remainder term is of higher order and $x > 0$, so

$$1 - \cos(x) \le \frac{x^2}{2} + \frac{x^3}{6}.\text{ ''}$$

4.b) The following argument is not wrong but incomplete. Which important observation is missing?

"Take $x = 1/n > 0$. Then, according to 4.a),

$$1 - \cos(1/n) \le \frac{1}{2n^2} + \frac{1}{6n^3} \le \frac{2}{3n^2}.$$

the series $\sum \frac{1}{n^2}$ is hyperharmonic and therefore convergent. So the series $\frac{2}{3}\sum \frac{1}{n^2}$ is a convergent majorant for the series $\sum(1 - \cos(1/n))$, so that one is convergent as well."

6.b) The following reasoning contains an error which is easily corrected. How?

"*. . . Choose* $x = \frac{1}{\sqrt{n}}$. *Then*

$$\|f_n\|_\infty = f_n(\frac{1}{\sqrt{n}}) = \frac{\sin 1}{2},$$

and therefore $\|f_n\|_\infty$ *does not approach zero as* $n \to \infty$. *So* (f_n) *not uniformly covergent on* $(0, \infty)$."

6.c) A similar error:

$$\text{“} \sup_{x\in[a,\infty)} |f_n(x)| = \left| \frac{\sin(na)}{1 + na^2} \right| \to 0$$

as $n \to \infty$, *and therefore* (f_n) *is uniformly convergent on* $[a, \infty)$. "

6.e) The following is not wrong but too short. What is the missing step?

"*. . . so the function* s *is continuous on the interval* $[a, \infty)$, *and therefore also on* $(0, \infty)$."

7. This hurts. Why, and how to do this better?

"*. . . therefore, for the coefficient of the power series we have*

$$a_0 = 1, \quad a_1 = a_2 = 0, \quad a_{k+3} = \frac{a_k}{(k + 1)(k + 2)(k + 3)}.$$

For the radius of convergence R *this implies*

$$R = \lim_{k\to\infty} \frac{1}{\left|\frac{a_{k+1}}{a_k}\right|} = \lim_{k\to\infty} \left|\frac{a_k}{a_{k+1}}\right| = \ldots \text{”}$$