1. First show that $\sup A + 2\sup B$ is an upper bound for C. Then show that C has no smaller upper bounds, i.e.:

$$\forall\, \varepsilon > 0 : \quad \exists\, c \in C : \quad c > \sup A + 2\sup B - \varepsilon.$$

   This is equivalent to

$$\forall\, \varepsilon > 0 : \quad \exists\, a \in A, b \in B : \quad a + 2b > \sup A + 2\sup B - \varepsilon.$$

2. Consider convergent subsequences (a_{n_k}) of (a_n) and the corresponding subsequences b_{n_k} with the same index sequence.

3. As far as convergence is concerned, we expect the series to behave like

$$\sum_{n=1}^{\infty} \frac{1}{2^{n/2}}, \qquad \sum_{n=1}^{\infty} \frac{1}{2^{2/n}}, \qquad \sum_{n=1}^{\infty} \frac{2}{n+4}.$$

   Find the behavior of these first. To prove results on the original ones, use (for example) convergent majorants or divergent minorants. (Other arguments are possible as well.)