1. (a) First show that $\sup A$ is an upper bound for $A \setminus \{z\}$. Then show that for all $\varepsilon > 0$, we have that $\sup A - \varepsilon$ is no upper bound for $A \setminus \{z\}$, i.e. that there is an element $a \in A \setminus \{z\}$ such that $a > \sup A - \varepsilon$.

   (b) Give an induction proof on the basis of a).

2. Use the squeeze theorem.

3. As far as convergence is concerned, we expect the series to behave like

$$\sum_{n=1}^{\infty} n^{-2}, \qquad \sum_{n=1}^{\infty} (-2)^{-n}.$$

   Find the behavior of these first. To prove results on the original ones, use (for example) convergent majorants or divergent minorants. (Other arguments are possible as well.)

4. Conclude first that $a_n > 0$ for n large. Intuitively, we expect that (a_n) "behaves like $1/n^2$". Use the limit relation to show a related estimate, and find a convergent majorant for $\sum a_n$.