Exercises Analysis 1 (2WA30) Lecture 2

1. Show: For all $a, b, c \in \mathbb{R}$ holds

a) $|a + b| \leq |a| + |b|$,

b) $|a - b| \leq |a - c| + |b - c|$,

c) $\big||a| - |b|\big| \leq |a - b|$. (so-called inverse triangle inequality)

2. Let $\{a_n\}$ be a sequence, $a^* \in \mathbb{R}$. Show:

$$a_n \to a^* \quad \Leftrightarrow \quad |a_n - a^*| \to 0.$$

3. Is the following proposition true? Give a proof or a counterexample.

> "Let (a_n), (b_n) be sequences with $a_n \to a^*$, $b_n \to b^*$, and $a_n < b_n$ for all n.
> Then $a^* < b^*$."

4. Let (a_n), (b_n),(c_n) be sequences defined by

$$a_n = \frac{n+3}{n^2-3}, \quad b_n = \frac{n^5}{3^n}, \quad c_n = \frac{7n+4}{n+5}.$$

Find the limits of these sequences and give proofs for the convergence towards these limits (without using limit theorems).