Opgaven Analyse 1 (2WA30) College 4

1. Find the limits of the following sequences:

$$a_n := \left(1 + \frac{1}{n}\right)^{2n}, \quad b_n := \left(\frac{n+8}{n+7}\right)^{n+4}, \quad c_n := \left(1 + \frac{1}{n^2}\right)^{n}, \quad d_n := \left(1 - \frac{1}{n}\right)^{n}$$

2. Show: Each sequence that is unbounded above / below has a subsequence that converges (improperly) to $+\infty$ / $-\infty$. Conclude that each sequence of real numbers has a properly or improperly convergent subsequence.

3. Find all accumulation points of the following sequences:

$$a_n := \frac{(-2)^n - 2n}{2^{n+1} + n^2}, \quad b_n := \sin\left(\frac{2\pi n}{3}\right) + \frac{1}{n}$$

(In particular, show that you found *all* accumulation points!)

4. Let (a_n) be a bounded sequence that has precisely the accumulation points 2 and 3. Show

$$\exists n_0 \in \mathbb{N} : \ \forall n \geq n_0 : \quad a_n \in (1, 4).$$

(Hint: Argue by contradiction and apply the Bolzano-Weierstrass theorem.)

5. For a sequence (a_n), let $V((a_n))$ be the set of its accumulation points. Give examples of sequences (a_n) such that:

- **a)** $V((a_n))$ has precisely four elements,
- **b)** $V((a_n))$ has infinitely many elements,
- **c)** $\star$ $V((a_n)) = [0, 1]$.

6. $\star$ Let (a_n) be a sequence. Let a^* be an accumulation point for each subsequence of (a_n). Show: $a_n \to a^*$.