Exercises Analysis 1 (2WA30) Lecture 10

1. Kosmala Ex. 8.1.1 a)-h)

2. Let $D \subseteq \mathbb{R}$, $f, g : D \longrightarrow \mathbb{R}$ be two bounded functions, $a \in \mathbb{R}$. Show:

$$\begin{aligned} \|f+g\|_\infty &\leq \|f\|_\infty + \|g\|_\infty, \\ \big|\, \|f\|_\infty - \|g\|_\infty \big| &\leq \|f-g\|_\infty, \\ \|af\|_\infty &= |a|\, \|f\|_\infty, \\ \|f\|_\infty = 0 &\Rightarrow (f \equiv 0). \end{aligned}$$

3. **a)** Give an example of a sequence of bounded functions (f_n) on $[0,1]$ which converges pointwise to an unbounded function f^*.

b) Let (f_n) be a sequence of functions on $D \subseteq \mathbb{R}$ with $f_n \to f^*$ uniformly on D. Assume f_n bounded for all n. Show that then f^* is bounded as well, and

$$\|f_n\|_\infty \to \|f^*\|_\infty, \quad \sup_{x\in D} f_n(x) \to \sup_{x \in D} f^*(x) \quad \text{as } n \to \infty.$$

4. Kosmala Ex 8.2.1. a) (with 8.1.1. a-h)

5. Kosmala Ex 8.2.2

6. Let $D \subset \mathbb{R}$ be unbounded from above and let (f_n) be a sequence of functions that convergence uniformly on D to some function f^*. Assume

$$\lim_{x\to\infty} f_n(x) = L_n \qquad \text{and} \qquad \lim_{n\to\infty} L_n = L^*.$$

Show:

$$\lim_{x\to\infty} f^*(x) = L^*.$$

(**Hint:** Modify the proof given in the lecture for limits where $x \to a \in D'$.)