Exercises Analysis 1 (2WA30) Lecture 13

1. Let $\{b_n\}$ be a sequence of real numbers. Show: If $\{b_n\}$ converges then

$$\limsup b_n = \lim_{n\to\infty} b_n.$$

2. Let $\{b_n\}$ be a sequence of numbers such that $b_n \geq 0$ and $\{c_n\}$ a sequence of numbers such that $c_n \to 1$. Show:

a)

$$\limsup(c_n b_n) = \limsup b_n,$$

b)

$$\limsup(b_n^{c_n}) = \limsup b_n.$$

Hint: Distinguish the cases $\limsup b_n = 0$ and $\limsup b_n > 0$. In the second case, use that whenever (x_k) is a sequence in $(0,\infty)$ with $x_k \to x^* > 0$ and (y_k) is a sequence in $\mathbb{R}$ with $y_k \to y^*$ then

$$x_k^{y_k} = e^{y_k \ln x_k} \to x^{*y^*}.$$

(Why?)

3. Kosmala Ex. 8.5.6

Hint: Ratio test.

4. Kosmala Ex. 8.5.3 (a)-(e) (A series is called conditionally convergent (dutch: voorwaardelijk convergent) if it is convergent but not absolutely convergent.)