Standard power series

All given series here are Taylor series around $x_0 = 0$.

Geometric and related series

$$\boxed{(1-x)^{-1} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \dots .}$$

From this we directly derive:

$$\begin{aligned}
(1 \mp x^m)^{-1} &= \sum_{k=0}^{\infty} (\pm 1)^k x^{mk} = 1 \pm x^m + x^{2m} \pm \dots, \\
\ln(1-x) &= -\int_0^x \frac{1}{1-t}\,dt = -\sum_{k=1}^{\infty} \frac{x^k}{k} = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots\right), \\
\arctan x &= \int_0^x \frac{1}{1+t^2}\,dt = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} \pm \dots
\end{aligned}$$

The radius of convergence of these series is 1.

Exponential and related series

$$\boxed{e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots .}$$

From this we directly derive:

Even / odd part:

$$\begin{aligned}
\cosh x &= \frac{e^x + e^{-x}}{2} = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \dots, \\
\sinh x &= \frac{e^x - e^{-x}}{2} \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{6} + \frac{x^5}{120} + \dots .
\end{aligned}$$

Real / imaginary part of e^{ix}:

$$\begin{aligned}
\cos x &= \mathrm{Re}(e^{ix}) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} \pm \dots, \\
\sin x &= \mathrm{Im}(e^{ix}) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} \pm \dots
\end{aligned}$$

These series converge for all $x \in \mathbb{R}$.

Binomial series:

$$\boxed{(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k = 1 + \alpha x + \frac{\alpha(\alpha-1)}{1\cdot 2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{1\cdot 2\cdot 3}x^3 + \dots}$$

met

$$\binom{\alpha}{k} := \frac{\alpha(\alpha-1)\dots(\alpha-k+1)}{k!}.$$

The radius of convergence of this series is 1.

For $\alpha \in \mathbb{N}_+$, the series reduces to a finite sum given by Newton's binomial theorem.

For $\alpha = -1$ we get the geometric series.

Trigonometric addition formulas

For $\alpha, \beta \in \mathbb{R}$ we have

$$e^{i(\alpha+\beta)} = e^{i\alpha}e^{i\beta},$$

so

$$\cos(\alpha+\beta) + i\sin(\alpha+\beta) = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta).$$

Carrying out the multiplication on the right and splitting into real and imaginary part yields

$$\begin{aligned} \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta, \\ \sin(\alpha+\beta) &= \cos\alpha\sin\beta + \sin\alpha\cos\beta. \end{aligned}$$