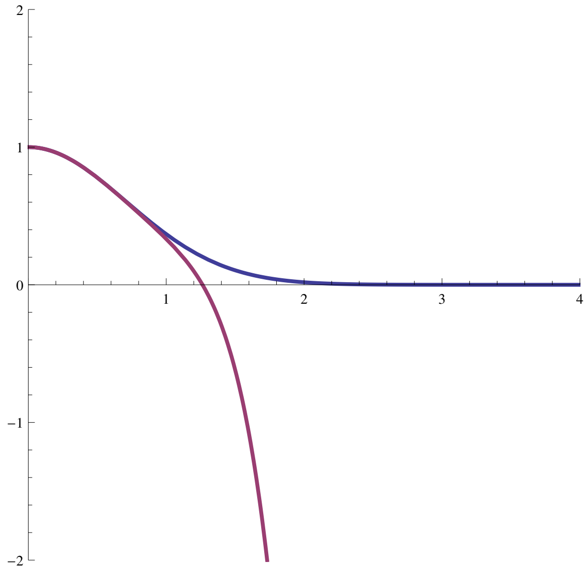
High order Taylor approximations

How does the Taylor polynomial $T_n = T_{n,f,x_0}$ behave as $n \to \infty$?

Example: $f(x) = e^{-x^2}$, $x_0 = 0$ $\quad \rightsquigarrow \quad$ $T_{2n+1}(x) = T_{2n}(x) = \sum_{k=0}^{n} \frac{(-x^2)^k}{k!}$ (check!)

$n = 2$:



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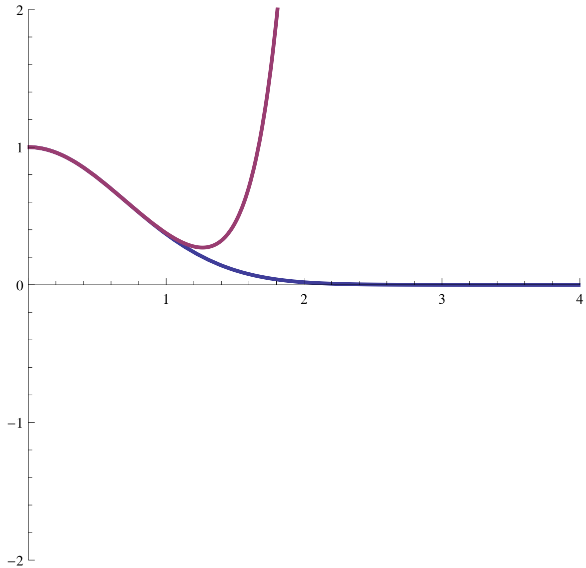

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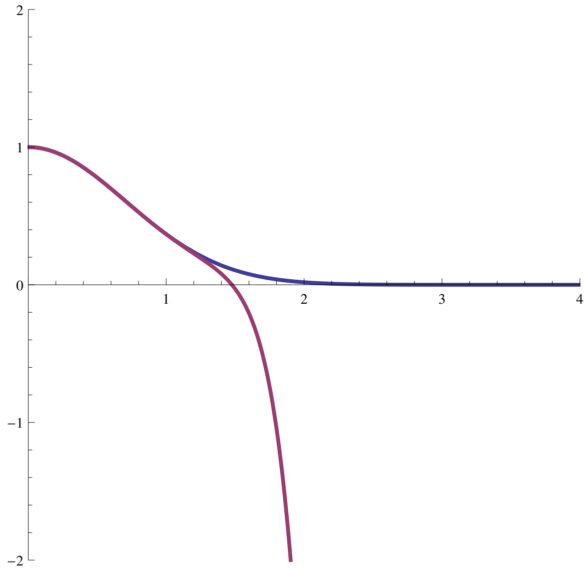

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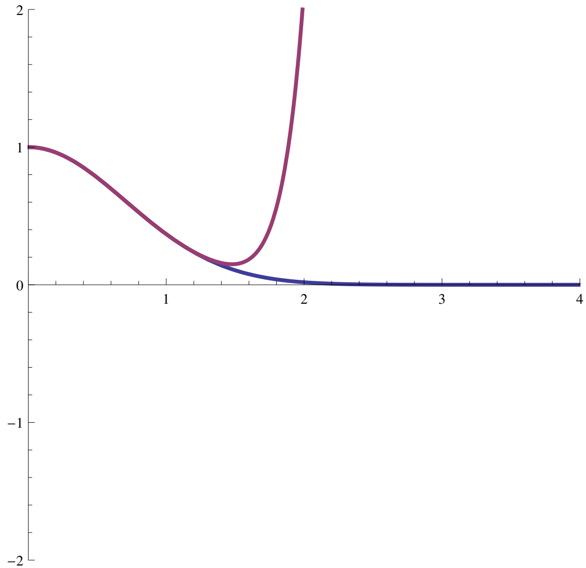

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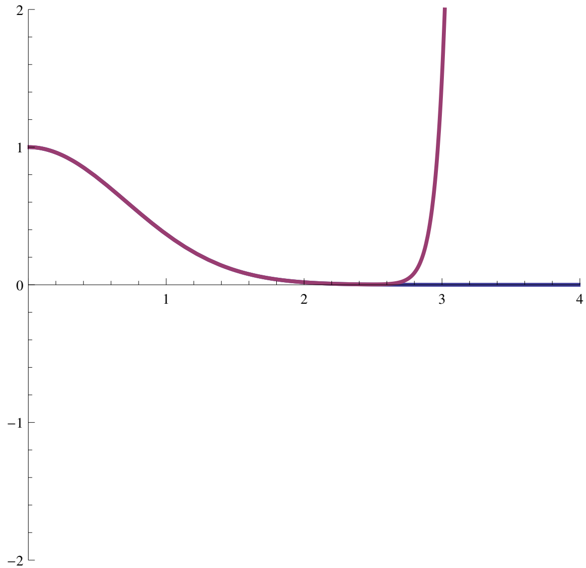

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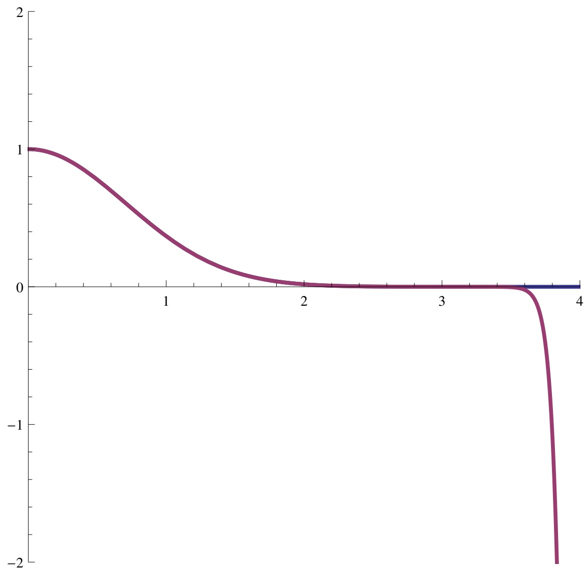

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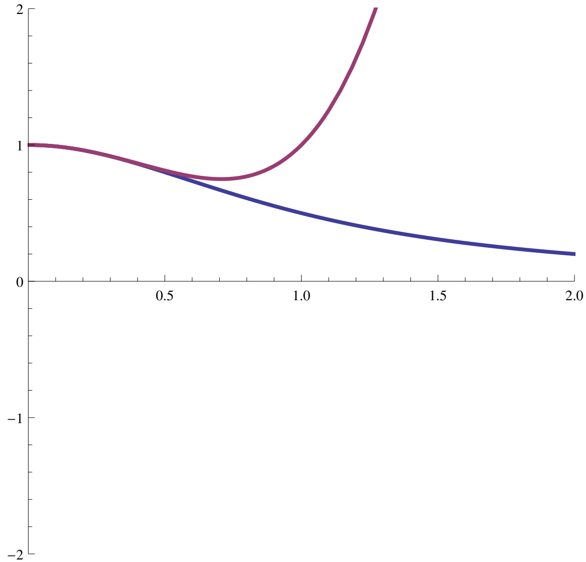

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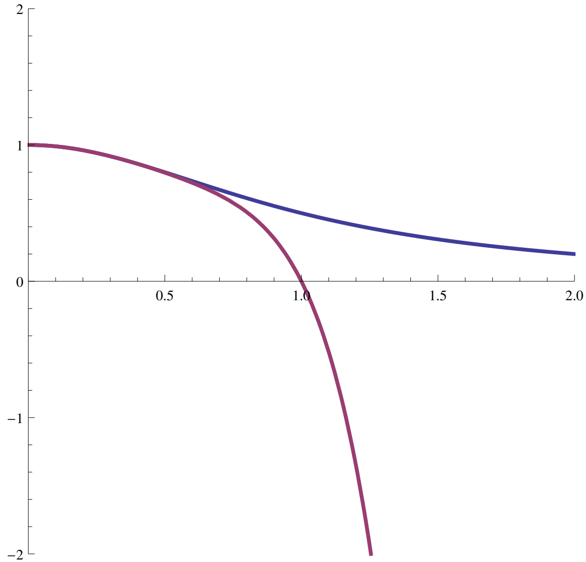

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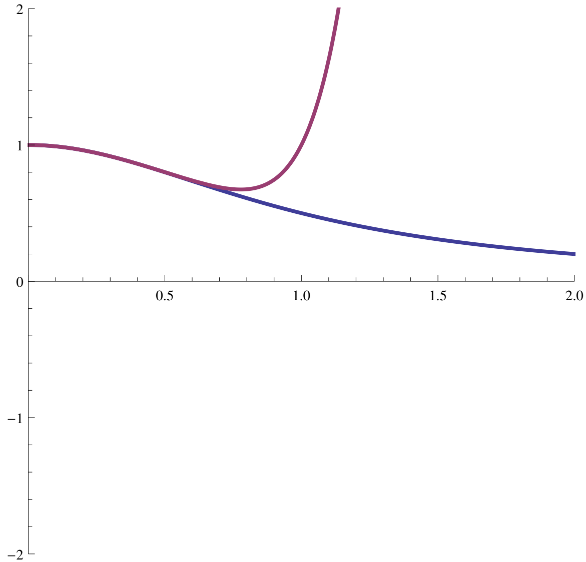

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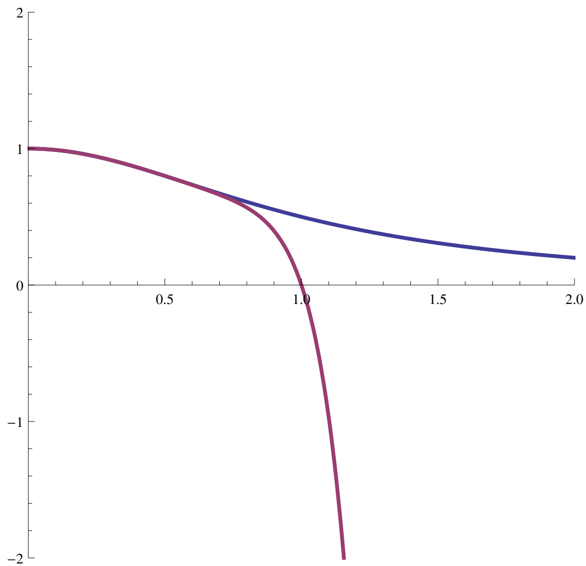

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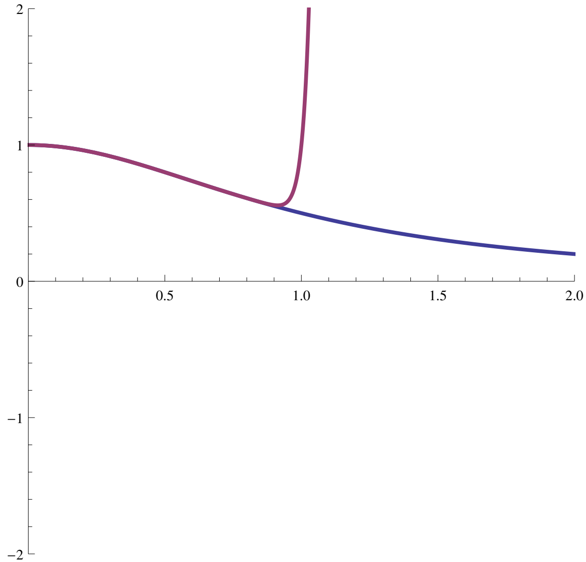

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