TECHNISCHE UNIVERSITEIT EINDHOVEN

Faculteit Wiskunde en Informatica

Intermediate test Analysis 1 (2WA32), wednesday 2 december 2015, 8.45-10.15 uur.

General instruction: Give reasons and arguments for all statements you make! Theorems that have been covered in the course may be used without reproving them. However, their use should be mentioned clearly and explicitly.

Use of any material (books, notes, calculators, computers etc.) is not allowed.

1. Let A, B be two bounded, nonempty subsets of $\mathbb{R}$. Let

$$C = \{a + 2b \,|\, a \in A,\, b \in B\}.$$

Show: $\sup C = \sup A + 2 \sup B$. (5 points)

2. Let (a_n) be a sequence of positive numbers having accumulation points 0 and 2. Let

$$b_n := \frac{a_n + n}{n a_n}, \qquad n \in \mathbb{N}_+.$$

a) Find an accumulation point of the sequence (b_n). (3 points)

b) Is the sequence (b_n) bounded? (3 points)

3. Let (a_n) be a sequence of positive numbers such that $a_n \to 2$ as $n \to \infty$. Determine whether the following series are convergent:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n^{n/2}}$ (3 points)

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n^{2/n}}$ (3 points)

c) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{(n + a_n^2)}$ (3 points)

Give reasons for your answers.

The mark is determined by dividing the total number of points by 2.