TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Intermediate test Analysis 1 (2WA32), wednesday 2 december 2015, 8.45-10.15 uur.

General instruction: Give reasons and arguments for all statements you make! Theorems that have been covered in the course may be used without reproving them. However, their use should be mentioned clearly and explicitly.

Use of any material (books, notes, calculators, computers etc.) is not allowed.

1. Let A be a nonempty, bounded subset of $\mathbb{R}$ with $\sup A \notin A$.

a) Let $z \in A$. Show: $\sup\big(A \setminus \{z\}\big) = \sup A$. (4 points)

b) Let $n \in \mathbb{N}_+$, $z_1, \ldots, z_n \in A$. Show: $\sup\big(A \setminus \{z_1, \ldots, z_n\}\big) = \sup A$. (2 points)

2. Let (c_n) be a bounded sequence in $\mathbb{R}$, and

$$\textbf{a) } a_n = \left(1 + \frac{1}{n + c_n}\right)^n, \qquad \textbf{b) } b_n = \left(1 + \frac{1}{n}\right)^{c_n}.$$

Determine whether (a_n) and (b_n) are convergent (for all bounded sequences (c_n)) and give the limit in this case. (2+2 points)

3. Let (a_n) be a sequence in $\mathbb{R} \setminus \{0\}$ with $\lim\limits_{n\to\infty} a_n = -2$. Determine whethere the following series converge (for all sequences (a_n) that satisfy the assumptions):

$$\textbf{a) } \sum_{n=1}^{\infty} n^{a_n}, \qquad \textbf{b) } \sum_{n=1}^{\infty} a_n^{-n}.$$

(3+3 points)

4. Is the following statement true? (Give a proof or a counterexample.)

"If (a_n) is a a sequence with $\lim\limits_{n\to\infty} n^2 a_n = 1$ then $\sum\limits_{n=1}^{\infty} a_n$ is convergent." (4 points)

The mark is determined by dividing the total number of points by 2.