EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Mathematics and Computer Science

Intermediate test Analysis 1, Wednesday December 5 2018, 8:45-10:15

Write the name of your instructor on the top of the first page you hand in!

General instruction: Give reasons and arguments for all statements you make! Theorems that have been covered in the course may be used without reproving them. However, their use should be mentioned clearly and explicitly.

Use of any material (books, notes, calculators, computers etc.) is not allowed.

1. Let A be a nonempty subset of the set of the real numbers. Let

$$B := \left\{ \frac{|a|}{1+|a|} \,\middle|\, a \in A \right\}.$$

Show: If A is unbounded then $\sup B = 1$. (6 POINTS)

2. Let (c_n) be a sequence in $\mathbb{R}$ such that for all $n \geq 2$ we have

$$\frac{1}{n+2018} \leq c_n \leq \frac{1}{n-1}$$

Does the sequence (x_n) given by

$$x_n = (1 + c_n)^n$$

converge? If yes, find the limit. (5 POINTS)

3. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{n^2}{n^4 - 1}$$

converges. (3 POINTS)

4. Let (a_n) be a sequence in $\mathbb{R}$ with $a_n > 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} a_n = 0$. Show: The sequence (a_n) has a subsequence (a_{n_k}) such that $\displaystyle\sum_{k=0}^{\infty} a_{n_k}$ is convergent. (6 POINTS)

The grade is determined by dividing the total number of points obtained by 2 and rounding to one decimal.