EINDHOVEN UNIVERSITY OF TECHNOLOGY
Department of Mathematics and Computer Science

Midterm test Analysis 1, Wednesday December 4 2019, 8:45-10:15

Write the name of your instructor on the top of the first page you hand in!

General instruction: Give reasons and arguments for all statements you make! Theorems that have been covered in the course may be used without reproving them. However, their use should be mentioned clearly and explicitly.

Use of any material (books, notes, calculators, computers etc.) is not allowed.

1. Are the following statements true? Give a proof or a counterexample:

a) "If A and B are two nonempty, bounded subsets of $\mathbb{R}$ such that $\sup A > \inf B$, then there is an $a \in A$ and a $b \in B$ such that $a > b$."

b) "If A and B are two nonempty, bounded subsets of $\mathbb{R}$ such that $\sup A > \inf B$ and $\sup B > \inf A$, then $A \cap B \neq \varnothing$."

(6 points)

2. Are the following statements true? Give a proof or a counterexample.

a) "For all sequences (x_n) in $(1, \infty)$ with $x_n \to 1$ we have

$$\frac{x_n}{x_n - 1} \to +\infty. \text{"}$$

b) "For all sequences (x_n) in $[1, \infty)$ with $x_n \to 1$ we have

$$\sqrt[n]{x_n} \to 1. \text{"}$$

c) "For all sequences (x_n) in $(0, \infty)$ with $x_n \to 1$ we have

$$x_n^n \to 1. \text{"}$$

(6 points)

3. Determine whether the following series converge or diverge, and prove your result.

$$\textbf{a)} \sum_{k=1}^{\infty} \left(\frac{k+1}{2k-1} \right)^k, \qquad \textbf{b)} \sum_{k=1}^{\infty} \frac{k + a_k}{k^2 + a_{k+1}},$$

where (a_k) is a bounded sequence of positive real numbers. (4+4 points)

The grade is determined by dividing the total number of points obtained by 2 and rounding to one decimal.