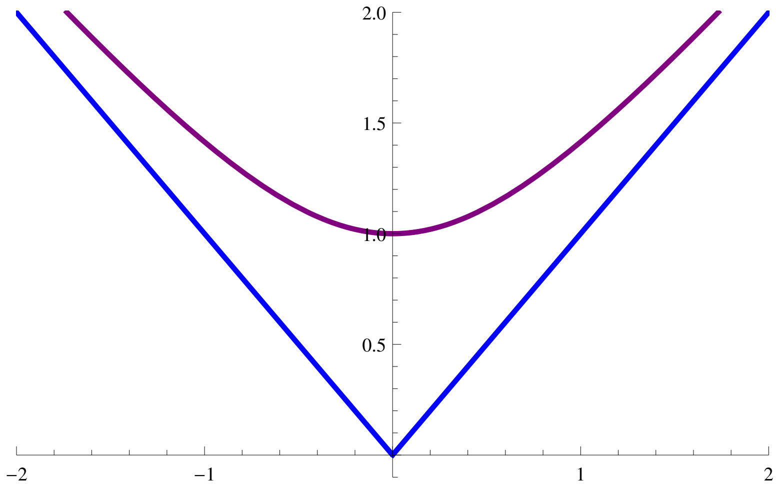
Uniform convergence does not preserve differentiability!

Example: $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, $\qquad f^*(x) = |x|$.

Then $f_n \to f^*$ uniformly on $\mathbb{R}$.

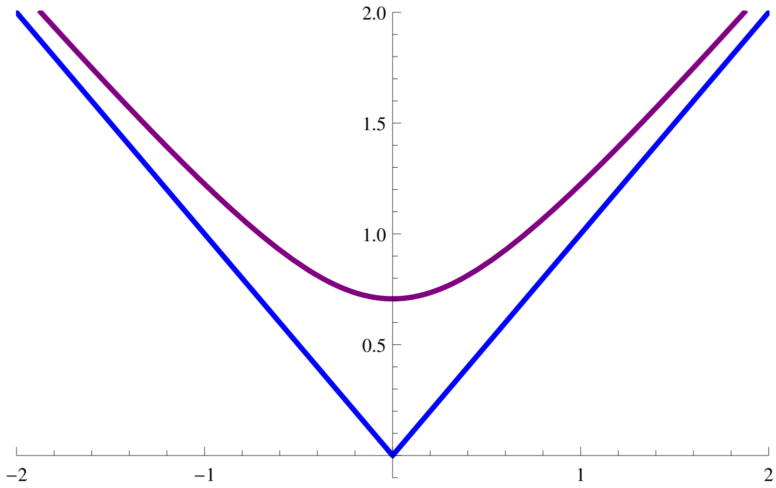
$n = 1:$



Uniform convergence does not preserve differentiability!

Example: $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, $\qquad f^*(x) = |x|$.
Then $f_n \to f^*$ uniformly on $\mathbb{R}$.

$n = 2:$

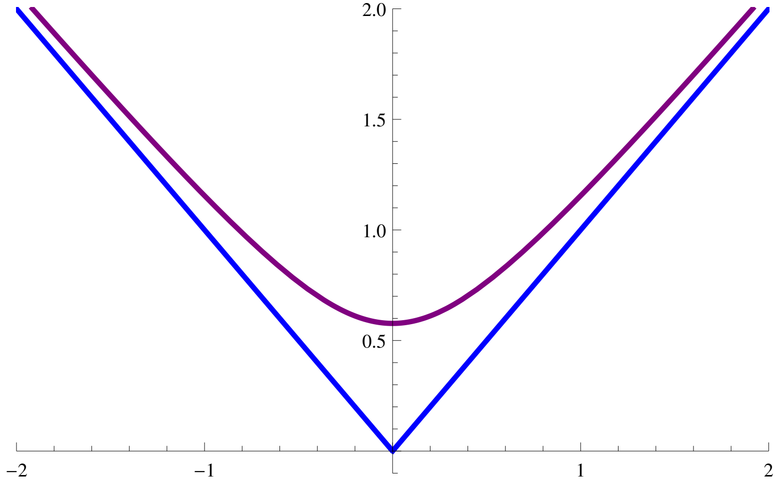


Uniform convergence does not preserve differentiability!

Example: $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, $\qquad f^*(x) = |x|$.

Then $f_n \to f^*$ uniformly on $\mathbb{R}$.

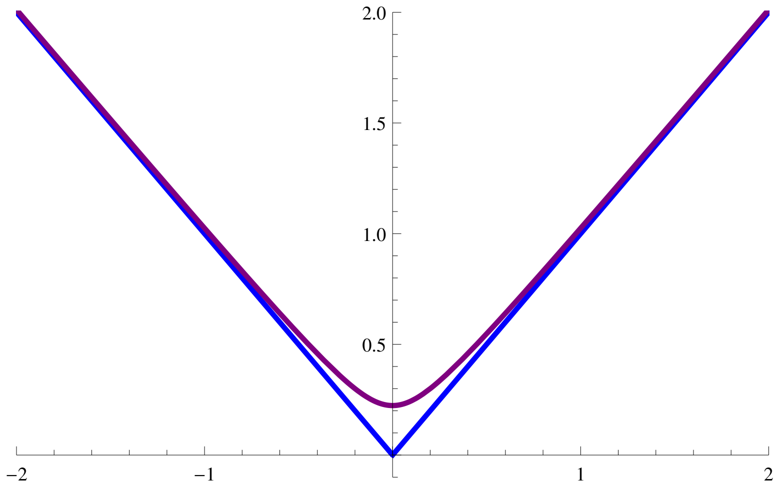
$n = 3:$



Uniform convergence does not preserve differentiability!

Example: $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, $\qquad f^*(x) = |x|$.
Then $f_n \to f^*$ uniformly on $\mathbb{R}$.

$n = 20:$

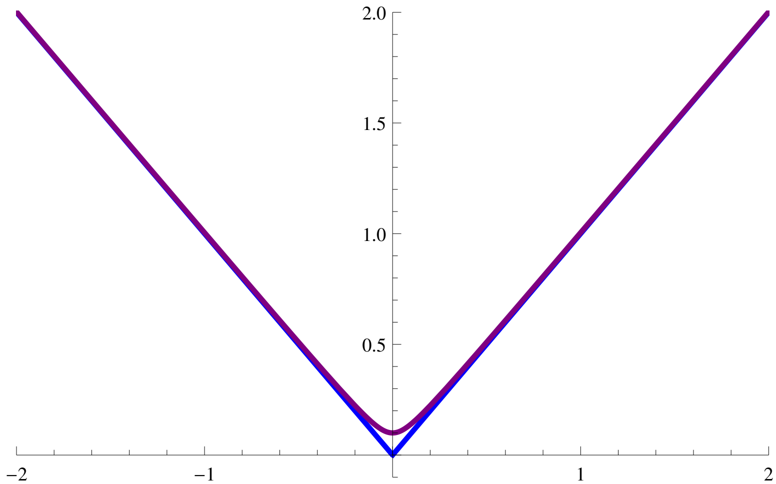


Uniform convergence does not preserve differentiability!

Example: $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, $\qquad f^*(x) = |x|$.

Then $f_n \to f^*$ uniformly on $\mathbb{R}$.

$n = 100:$



Uniform convergence does not preserve differentiability!

Example: $f_n(x) = \sqrt{x^2 + \frac{1}{n}}$, $\qquad f^*(x) = |x|$.
Then $f_n \to f^*$ uniformly on $\mathbb{R}$.

$n = 100:$