Optimization (2MMD10/2DME20), Final week

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General information

Exam (2MMD10 and 2DME20):
Friday, October 30, 09:00–12:00

Second chance exam:
January 27 (2MMD10) and April 8 (2DME20)

- The exam consists of 8 questions

- English okay; Dutch okay
- Write clearly!
- Start every question on a new page

Types of exam questions

Typical types of exam questions:

- Formulate a result or definition presented in the course.
  Formulate a result or definition from the reading homework.
- Show a proof (or proof piece) presented in the course.
  Show a proof (or proof piece) from the reading homework.
- Construct a solution by applying a known theorem.
- Apply a known theorem to a concrete scenario.
- Decide whether some simple statement is true. (Show argument!)
- Provide a proof for a new statement.

Problem 1

Let $f : \mathbb{R}^+ \to \mathbb{R}^+$ be a convex function.
Does this imply that the function $xf(x)$ is convex on $\mathbb{R}^+$?

Typical student answers:

Yes!

No!

Yes and No.

Yes or No.

Problem 1

Let $f : \mathbb{R}^+ \to \mathbb{R}^+$ be a convex function.
Does this imply that the function $xf(x)$ is convex on $\mathbb{R}^+$?

Typical student answers:

Yes. A theorem from the course says that the product of two convex functions is again convex. As $f(x)$ is convex and $g(x) = x$ is linear, $f(x)g(x) = xf(x)$ is also convex.

Yes. The first derivative of $xf(x)$ is $f(x) + xf'(x)$. The second derivative of $xf(x)$ is $2f'(x) + xf''(x)$.

No. The first derivative of $xf(x)$ is $f(x) + xf'(x)$. The second derivative of $xf(x)$ is $2f'(x) + xf''(x)$.

Problem 1a

Decide whether the function $f : \mathbb{R}^2_+ \to \mathbb{R}$ with $f(x, y) = (x - y)^2/(2x + y)$ is convex.

Problem 2

An instance of the EXACT-PATH problem consists of an undirected graph $G = (V, E)$ together with a weight function $w : E \to \mathbb{Z}$, two vertices $s, t \in V$, and a value $U \in \mathbb{Z}$. The question is whether the graph contains a simple path from s to t so that the sum of edge weights along the path equals U.

(a) Prove that EXACT-PATH lies in NP.

(b) Prove that EXACT-PATH is NP-hard.

Problem 3

Give the Lagrangian, Lagrange dual function, and Lagrange dual of the problem $\min\{\sum_{i=1}^{3} x_i \log(x_i)\} \mid x_1 + x_2 + x_3 = 1,\ x_1 + 2x_2 + 4x_3 \leq 2\}$. Is this problem convex?