Review for the AMS

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TITLE: Complementary to Yannakakis' theorem.
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M. Yannakakis [1] proved that the Traveling Salesman Problem (TSP) polytope cannot be expressed by symmetric polynomial size linear programs, where symmetry means that the polytope is invariant under vertex relabeling. In this paper the author associates to every instance of the more general Asymmetric Traveling Salesman Problem (ATSP) an asymmetric polynomial size linear program, and claims that this linear program faithfully represents the ATSP instance to which it has been associated. It is worth pointing out at this point that if this result were true then it would imply that P=NP. Let us also recall that, for a correct solution of the "P versus NP" question, the Clay Mathematics Institute (CMI) will award a prize of \$1.000.000. More information on this prize is available at HTTP://WWW.CLAYMATH.ORG/MILLENNIUM/. Up to now (18 Jan 2010) the prize is still there, despite various claims of a solution of this fundamental problem have been done. For an updated account on the history of these claims and their refutation see HTTP://WWW.WIN.TUE.NL/G̃WOEGI/P-VERSUS-NP.HTM. For example, the following item occurs in a list offered at this web page:

33. **[Equal]:** In October 2006, Sergey Gubin proved P=NP by constructing a polynomial time algorithm for the directed Hamiltonian cycle problem. His paper is available at HTTP://ARXIV.ORG/ABS/CS.DM/0610042. The title of the paper is "A Polynomial Time Algorithm for The Traveling Salesman Problem". ⟨Here⟩ are some comments by Radoslaw Hofman on this proof. And ⟨here⟩ is a full refutation of Gubin's arguments by Ian Christopher, Dennis Huo, Bryan Jacobs from April 2008. (Thanks to Juergen Ernst for providing the link to Christopher, Huo, and Jacobs.)

Since at present no refutation has yet been posted or published for the claims in the paper "Complementary to Yannakakis' theorem" and since JCMCC, contrary to the arxiv system, is actually a refereed scientific journal, let us point out a few serious problems which arise already with the first notions introduced within the paper addressed by the present review. For each two vertices i and j of the input digraph, the box is defined as the following matrix $C_{ij} = (c_{ih\mu\nu})_{n\times n}$:

$$c_{ij\mu\nu} = \begin{cases} 1 & s_{ij} \leq g_{\mu\nu} \wedge c_{ji} \leq g_{\nu\mu} \\ 0 & s_{ij} > g_{\mu\nu} \vee s_{ji} > g_{\nu\mu} \end{cases}$$

where G is the adjacency matrix of the input digraph and S is the adjacency matrix of a directed Hamiltonian cycle which visits the nodes $1, 2, \ldots, n$ following the order. Notice that there are problems with the above definition since c_{ji} is a matrix and not a scalar. In order for the above definition to be well posed, the two cases should partition the space of the possibilities, which by necessity leads to correct the likely typo as follows:

$$c_{ij\mu\nu} = \begin{cases} 1 & s_{ij} \leq g_{\mu\nu} \wedge s_{ji} \leq g_{\nu\mu} \\ 0 & s_{ij} > g_{\mu\nu} \vee s_{ji} > g_{\nu\mu} \end{cases}$$

However, the following two ”major properties” in the displayed Formula (1.3) do not hold:

1. it is not true that $C_{ji} = I_{n \times n}$;
2. it is not true that $i \neq j \Rightarrow c_{ij\mu\mu} = 0$.