Feedback in Learning to Solve Linear Algebra Problems: Effects on Efficiency and Motivation

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Abstract

In mathematical problems an error made in one step of the problem-solving procedure will carry over to the following steps and consequently to the final solution. Providing immediate feedback after an error is made could prevent such carry-over effects. This study investigated the learning and motivational effects of feedback on the problem-solving steps. Feedback given on all problem-solving steps was hypothesized to yield higher motivation and better learning than feedback on the final problem-solving step (i.e., solution step). A first explorative study investigated students’ perceptions of three feedback types: (A) ‘on the final solution step’; (B) ‘on all the solution steps at once’; and (C) ‘on all the solution steps successively’. Results showed that feedback on all solutions steps was perceived more positively than feedback on the final solution step. A second study investigated the learning and motivational effects of two types of feedback, namely, ‘feedback on the final solution step’ and ‘feedback on all the solution steps’ (a combination of feedback B and C in the exploratory study). The hypothesis that feedback on all problem-solving steps would lead to more efficient learning and higher motivation than feedback on the final solution step was confirmed. The implications for the design of feedback are discussed.

Keywords: efficiency; feedback; motivation; problem-solving steps
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1. Introduction

Binnen technische en beta-studies zijn wiskundige kennis en vaardigheden veelal noodzakelijk voor het oplossen van problemen. Many mathematical problems require students to apply a standard problem-solving procedure. If the student makes an error while solving such a problem, the subsequent solution steps, and consequently the final solution, can be incorrect because the error will carry over to the following solution steps (Moreno, Reisslein, and Delgoda, 2006). To prevent this carry-over effect in the acquisition of mathematical problem-solving skills, students can be best provided with feedback immediately after an error has been made. This can only be realized if the error made by the student can be detected during the problem-solving process and by providing specific and timely feedback to that error. However, it is nearly impossible for teachers to provide such feedback in an individualized format to more than one student simultaneously (Baki and Güveli, 2008). From that perspective, teacher provided feedback may negatively influence students’ learning, and consequently their motivation to continue learning. The National Council of Teachers of Mathematics\(^1\) (2000) report emphasizes the importance of technology for teaching and learning mathematics. Nevertheless, whereas there are many electronic tools that provide feedback after the final solution is provided by the students, tools that allow students to solve a problem in a step-wise fashion and that provide feedback after each step have rarely been investigated (Jeuring and Pasman, 2007).

The goal of the present study was to investigate the motivational and learning effects in freshman computer science and mathematics students who studied linear algebra problems with different types of automatically generated feedback. More specifically, this study aimed at determining if feedback on all the problem-solving steps yields more beneficial effects for learning and motivation than feedback on the final problem-solving step. The next sections

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\(^1\) For more information visit the URL [http://standards.nctm.org/](http://standards.nctm.org/)
describe how feedback on the problem-solving steps can enhance students’ learning and motivation.

1.1. Feedback and learning

Feedback provides students with information about their learning and achievement (Butler and Winne, 1995) and gives learners the opportunity to adjust and develop their cognitive strategies and to rectify misconceptions while progressing through the training (Azevedo and Bernard, 1995). Feedback has been argued to play an important role in learning (Hattie and Timperley, 2007; Mory, 2003) and to influence performance in different ways depending on how it is provided (Moreno, 2004). Especially during initial practice, feedback should be provided during each step of the problem-solving procedure (see Mory, 2003). Feedback on all the problem-solving steps allows learners to immediately verify the correctness of a solution step while the corresponding step is still in working memory (Moreno et al., 2006) and enables learners to focus their attention on the solution steps (Mory, 2003).

Particularly in the mathematics domain, feedback on all the problem-solving steps supports the learners to identify where (in which step) in the process something went wrong. A student may solve an equation problem in a number of steps. For example, a system of linear equations can be solved by Gaussian elimination, a procedure that starts with subtracting equations from top to bottom, and then solving for the various unknowns subtracting variables from bottom to top (Jeuring and Pasman, 2007; Passier and Jeuring, 2006). Gaussian elimination can be found in any standard textbook on Linear Algebra or Matrix theory (e.g., Kolman and Hill, 2006; Lay, 2002). Providing feedback on the final problem-solving step does not support learners to identify where exactly the problem was, with obvious negative effects on learning. An electronic tool that recognizes the error made by the student while solving the problem can provide specific feedback on that step.
1.1.1. Cognitive load

Although test performance results have typically been used to assess learning outcomes, Cognitive Load theory (CLT: Paas, Renkl, and Sweller, 2003; Paas, Tuovinen, Tabbers, and van Gerven, 2003; Sweller, van Merriënboer, and Paas, 1998) argues that the imposed cognitive load to attain this performance should be considered as well. Cognitive load is commonly defined as the learner’s cognitive capacity that is allocated to accommodate the demands imposed by the task, which is measured as the amount of mental effort invested by the learners to perform the task. Paas and van Merriënboer (1993; see also Van Gog and Paas, 2008) have introduced an approach to combine measures of performance and mental effort to get a reliable estimate of the relative efficiency of instructional conditions. According to this approach, efficiency is high if high performance is combined with low mental effort, and efficiency is low if low performance is combined with high mental effort.

Cognitive Load Theory distinguishes between three types of cognitive load: intrinsic, extraneous and germane. According to Sweller et al. (1998) intrinsic cognitive load relates to the nature of the learning material and is caused by the number of interacting information elements in a task. Besides the load imposed by the task, there is also load that is imposed by the instructional design. When this load is ineffective for learning, it is called extraneous cognitive load, and when it is effective for learning it is referred to as germane cognitive load (Sweller et al., 1998). From a cognitive load perspective, feedback on all the problem-solving steps reduces extraneous or ineffective cognitive load that might be caused when students are provided with feedback on the final solution step only and have to search for a plausible explanation to the correctness or incorrectness of their solution (Moreno, 2004). Such inefficient strategies result in high cognitive load and low learning outcomes. Whereas many studies have reported efficiency measures (for an overview, see Paas et al., 2003), there seem
to be only three studies (Corbalan, Kester, and van Merriënboer, in press; Halabi, 2006; and Moreno, 2004) that measured the effects of feedback on efficiency. However, feedback was operationalized differently and the domains used were different from the ones used in the present study.

1.2. Feedback and motivation

The motivational effects related to feedback have been recognized by many authors (e.g., Azevedo and Bernard, 1995; Chai, 2003; Hyland, 2001; Keller, 1983; Mory, 2003; Ross and Morrison, 1993; Vollmeyer and Rheinberg, 2005). Feedback on the problem-solving process helps learners focus on the solution steps and enables them to see the connection between what they need to learn and the learning opportunities presented to them (Keller, 1983, 1987). The cognitive effects of motivation result primarily from the relevance of what is being learned since relevance increases the use of cognitive strategies, which will eventually improve learning (Means, Jonassen, and Dwyer, 1997). In a review of studies on feedback, Hattie and Timperley (2007) noted that ‘whether students engage in error correction strategies following error detection depends on their motivation to continue to pursue the goal or to reduce the gap between current knowledge and the goal’ (p. 93). That is, transfer will be enhanced not so much by what is being taught, but by what learners are motivated to learn themselves (Berge and Collins, 1995).

1.3. The studies

Two studies investigated the effects of different feedback types on students’ perceptions (study 1) and on learning and motivation (study 2). The first study explored students’ perceptions regarding three feedback types, namely (A) ‘on the final solution step’, which provided feedback on the final solution only without providing any information to as whether the
intermediate solution steps are correct or not; (B) ‘on all the solution steps at once’, which consisted of a worked example providing learners with the problem statement, all the solution steps as well as the final solution; and (C) ‘on all the solution steps successively’, which was a step-by-step feedback providing learners with feedback after each solution step was solved. The second study measured the effectiveness (i.e., transfer performance), efficiency (i.e., transfer performance combined with mental effort invested to attain that performance) and motivational effects of two feedback types, namely, ‘feedback on the final solution step’ and ‘feedback on all the solution steps’ (a combination of feedback B and C in the first study). Feedback on all the solution steps are believed to support the learners to identify where, that is, in which step a (potential) problem is located. In addition, more basic feedback, such as feedback on the final solution step only, does not add sufficiently to schema development, and results in a higher mental effort to interpret it (Halabi, 2006). It is hypothesized that participants who are provided with feedback on all the steps of the problem-solving process will show higher effectiveness and higher efficiency than participants who are provided with feedback on the final solution step only. In addition, feedback on all the problem-solving steps is expected to yield higher motivation, because it enables learners to see the connection between what they need to learn and the learning opportunities presented to them.

Experiment 1

Method

Participants

Nine first-year university students of the Technical University Eindhoven (2 females and 7 males; mean age = 18.11 years; $SD = .60$) participated in this exploratory study. All participants spoke Dutch as their first language, the language in which the instruction was given. They received €50 (approximately $75) for their participation.

Materials and Measurements
Electronic learning environment. Training problems were designed and delivered with Wortel TU/e\(^2\), the electronic learning environment for mathematics available at the Technical University Eindhoven. The learning environment contained three sets of problems in linear algebra. Each set comprised three problems, increasing in difficulty, and included one of the three feedback types: (A) ‘on the final solution step’, which provided information on whether the final solution was correct or wrong, followed by the correct solution; (B) ‘on all the solution steps at once’, which consisted of a worked example showing all the solution steps at the same time; and (C) ‘on all the solution steps successively’, which was a step-by-step feedback which provided a step-by-step guide to solve the problem. A within-subjects design was used in which all the participants received the three available feedback types. To avoid order effects, the order in which the sets and its related feedback type were provided to participants was counterbalanced.

*Feedback specific questionnaire.* The questionnaire consisted of: (a) a perception questionnaire which comprised ten statements in relation to the feedback provided and it used a 5-point Likert scale ranging from ‘1 strongly disagree’ to ‘5 strongly agree’; and (b) an overall usefulness question (also measured with a 5-point Likert scale). Because this questionnaire was feedback specific, each participant filled out the questionnaire three times, one time per feedback type. Reliabilities of the perception questionnaire for the three types of feedback were .93, .90, and .83 (Cronbach’s alpha) for, in order, ‘feedback 1’, ‘feedback 2’, and ‘feedback 3’.

*Feedback general questionnaire.* This questionnaire consisted on two sets of questions. Set 1 contained eight multiple-choice questions which required participants to choose one of the three feedback types (e.g., *Which of the three types of feedback provided do you prefer the most?*; *Which of the three types of feedback was the least informative?*). For the exact wording

\(^2\) For more information visit the URL [http://wortel.tue.nl](http://wortel.tue.nl)
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of each question, the reader is referred to Table 2. Set 2 contained three questions in which participants had to indicate their preferred type of feedback given an easy, a medium, and a difficult linear algebra problem, respectively.

Procedure

First, one of the two experimenters explained the purpose of the session to the participants. Subsequently, participants worked with the three sets of linear algebra problems. After each set of problems, participants filled out the ‘feedback specific questionnaire’, which measured their perceptions regarding the specific feedback type they just received. Each set of problems took between 15 and 20 minutes to complete. After the three sets of problems were completed, participants filled out the ‘feedback general questionnaire’.

Results

Feedback specific. Table 1 presents the descriptive results of the perception scale and the overall usefulness question. Perception scores are an average of the scores of the ten statements measured with the 5-point Likert scale. A nonparametric Friedman’s test revealed that participants’ perception was significantly different between the three types of feedback ($X^2 (2) = 10.89, p < .01$). Wilcoxon tests were used to follow up this finding. After a Bonferroni correction, all effects are reported at a .0167 level of significance. It appeared that participants’ perception was significantly different between the feedback ‘on the final solution step’ and the feedback ‘on all the solution steps at once’ ($Z = -2.67, p = .01$) and between the feedback ‘on the final solution step’ and the feedback ‘on all the solution steps successively’ ($Z = -2.25, p = .015$). Similarly, participants’ perceived overall usefulness was significantly different ($X^2 (2) = 8.06, p < .025$). Follow up Wilcoxon tests revealed significant differences between the perceived overall usefulness of students between the feedback ‘on the
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final solution step’ and the feedback ‘on all the solution steps at once’ \((Z = -2.40, p = .01)\). No other significant effects were found.

[Insert Table 1 about here]

Feedback general. Table 2 presents the students’ selected feedback types to each question of the two sets of questions.

[Insert Table 2 about here]

Chi-square goodness-of-fit were used to analyse the results. Regarding the first set of questions, results show a significant difference between the three feedback types on ‘question 1’ \((X^2 (2) = 8.67, p = .013)\); ‘question 2’ \((X^2 (2) = 8.67, p = .013)\); ‘question 6’ \((X^2 (2) = 18, p = .000)\); and ‘question 8’ \((X^2 (2) = 12.67, p = .002)\). Additionally, marginal significant differences were found on ‘question 4’ \((X^2 (2) = 6, p = .05)\); ‘question 5’ \((X^2 (2) = 6, p = .05)\); and ‘question 7’ \((X^2 (2) = 6, p = .05)\). With regard to the second set of questions, which required participants to select their preferred feedback type given a specific problem with low, medium, and high difficulty, a marginal significant effect was found on ‘question 3’ \((X^2 (2) = 6, p = .05)\). Figure 1 depicts students’ preferred feedback type per difficult level.

[Insert Figure 1 about here]

Conclusions

The current study gives insight into students’ perceptions regarding three different types of feedback in linear algebra problems, namely, feedback ‘on the final solution step’ and two feedback on all the solution steps (i.e., feedback ‘on all the solution steps at once’ and feedback ‘on all the solution steps successively’). Students’ perceptions were clearly more positive for the two feedback types which provided feedback on all the solution steps as compared to the feedback provided on the final solution step only. Consequently, the next study investigated the motivational and learning effects of a feedback condition in which the two types of feedback on all the solution steps were combined, as compared to a feedback
condition in which only feedback on the final solution step was provided. None of the students selected feedback ‘on the final solution step’ as the most preferred, the easier to follow, the most informative, or as the type of feedback that they would choose for their own learning. In addition, none of the participants selected feedback ‘on the final solution step’ for solving the most difficult linear algebra problem. The degree to which a specific feedback type is preferred to solve linear algebra problems seems to depend on the level of difficulty of the exercise to solve. It appears that students prefer more detailed feedback as the level of difficulty of the problems increases.

Although this study aimed at exploring students’ perceptions of feedback, a replication of this study, which would include measures of students’ performance as well, could shed more light on the inconsistent results found in feedback studies. Forms of feedback which contain simple but sufficient information may help minimizing the associated cognitive load to process the information, because it might contain considerable less distracting information (for a review see Mory, 2003). According to this premise and to students’ selections, for simple problems a more basic feedback form (e.g., feedback ‘on the final solution step’) may prove sufficient. In sum, these findings show that in general, students clearly prefer feedback on all the solution steps to feedback on the final solution step only.

**Experiment 2**

**Method**

*Participants*

Thirty-four university students of the Technical University Eindhoven (5 females and 29 males; mean age = 20.29 years; $SD = 2.98$) participated in this study. All participants spoke Dutch as their first language, the language in which the instruction was given. They received €100 (approximately $150) for their participation. The students were randomly assigned to one of the two experimental groups, which were decided on the basis of the results
of the exploratory study, as stated above. In the feedback ‘on the final solution step’ condition \((n = 17)\), the participants were provided information on whether their answer was correct or wrong, that is, they were provided with feedback for the final solution only. In the feedback ‘on all the solution steps’ condition \((n = 17)\) the participants received feedback on the each solution step at once (worked example) or successively (step-by-step) and was a combination of the two feedback on all the solution steps from the exploratory study.

**Materials and Measurements**

*Training problems.* Similar to the first study, training problems were designed and delivered with Wortel TU/e. The learning environment contained two sets of problems in linear algebra. Each participant worked on the first set during a first session, and on the second set during a second session. The first set contained ten problems regarding the problem of solving a set of linear equations and on subspaces, basis and dimension of linear space (in Dutch abbreviated to, vergelijkingen en bases). The second set contained ten problems regarding matrices and linear transformations (in Dutch, matrices en afbeeldingen). Figure 2 shows an example of a training task provided to the participants in the feedback ‘on the final solution step’ condition during the first session (Figure 2a) and an example of a training task provided to the participants in the feedback ’on all the solution steps’ condition during the second session (Figure 2b). Readers interested in getting more insight into the type and content of the training tasks presented to the students are referred to Cuypers (2007)\(^3\), who describes a number of rules and strategies to solve exercises in the area of basic linear algebra.

*Test problems.* Test tasks were delivered on paper. In total, there were eight problems, more specifically, 4 near transfer tasks and 4 far transfer tasks. The near transfer tasks were structurally similar to the training tasks (see Figure 3a for an example) and determined

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whether participants were able to apply the learned procedures in the same way as in the training tasks. The far transfer tasks required participants to flexibly apply the learned solutions procedures to tasks with different structural features than those encountered during the training sessions (see Figure 3b for an example). For example, during the transfer test students encountered various tasks for finding the nullspace of a 2x2 or 3x3 matrix. Among the far transfer tasks we used the following problem:

'given three differentiable real functions \( f, g \) and \( h \) with derivatives satisfying

\[
\begin{align*}
  f' &= f + g + h, \\
  g' &= 2g + h, \text{ and} \\
  h' &= 2f - 2g,
\end{align*}
\]

is it possible to write the function \( k \) with \( k(x) = 1 \) for all real \( x \) as a linear combination of \( f, g \) and \( h \),'

Here the student had to realize, that taking derivatives on the space of differentiable functions that are linear combinations of \( f, g \) and \( h \) is a linear mapping that can be represented by a 3x3 matrix. The function \( k \) can be written as a linear combination of \( f, g \) and \( h \) if and only if the matrix has a nontrivial vector in its nullspace.

Mental effort. Mental effort was used as an index for cognitive load, which refers to the amount of cognitive capacity that is allocated to problem solving. Mental effort was measured as the “effort required to solve the task” (Paas, 1992; Paas et al., 2003) after each learning task and after each transfer task with a one-item 9-point Likert-type scale ranging from 1 (very very low effort) to 9 (very very high effort). Reliability of the mental effort measures reported during the training was .89 (Cronbach’s alpha) and during the transfer test .72 and .73 (Cronbach’s alpha) for the near and far transfer test, respectively.
Efficiency. Participants’ test performance on near and far transfer and mental effort invested to achieve this performance were combined using the procedure of Paas and van Merriënboer (1993) to calculate efficiency (E) on near and far transfer. Performance and mental effort scores are first standardized, and then the z-scores are entered into the formula:

\[ E = \frac{Z_{\text{Performance}} - Z_{\text{MentalEffort}}}{\sqrt{2}} \]

When performance is higher than might be expected based on perceived mental effort, the instructional condition is more efficient. Conversely, when performance is lower than might be expected based on perceived mental effort, the instructional condition is less efficient.

Motivation questionnaire. The Instructional Materials Motivation Survey (IMMS; Keller, 1983) assesses the motivational effects of instructional situations. Specifically, it asks students to rate 36 statements tapping attention (i.e., need for stimulation and variety), relevance (i.e., desire to satisfy basic motives), confidence (i.e., desire to feel competent and in control), and satisfaction (i.e., desire to feel good about oneself), about the learning materials according to the ARCS model (Keller, 1983a) with four respective subscales, namely Attention (e.g., “There was something interesting at the beginning of each task that got my attention”), Relevance (e.g., “It was clear to me how the content of the tasks was related to things I already know”), Confidence (e.g., “As I worked on the tasks, I was confident that I could learn the content”), and Satisfaction (e.g., “Completing the tasks gave me a satisfying feeling of accomplishment”). The scales contained, in order, twelve, nine, nine, and six items and were measured with a 5-point rating scale ranging from 1 (completely not true) to 5 (completely true). The reliability of the motivation questionnaire was .78 (Cronbach’s alpha).

Perception questionnaire. In addition to the motivation questionnaire and to compare students’ perceptions of the feedback provided with the results of the first study, participants received a similar questionnaire to the feedback specific questionnaire presented in the
explorative study. In addition, it included two “yes / no” questions which asked participants whether they ‘would like to see some features added to this type of feedback’ and whether they ‘would like to see some features removed from this type of feedback’. Reliability of the perception questionnaire was .93 (Cronbach’s alpha).

Procedure

First, one of the experimenters explained the purpose of the experimental sessions to the participants. They were explained that they would have to work during three different sessions: During each of the two first sessions, they would have to work on the computer on a set containing 10 types of linear algebra problems, and that for each problem feedback would be provided. They were also instructed that during the third session they would have to work on a post test containing eight paper and pencil linear algebra problems and that no further feedback would be provided. Subsequently, participants filled out a question on their prior knowledge in linear algebra and started the first session. The session took a maximum of two hours. Within the same week, participants worked on the second session for a maximum of two hours. After each training problem, mental effort was measured by asking the participants to fill out the 9-point rating scale. Immediately after the second session, participants filled out the perception and the motivation questionnaires. During the training sessions, the time spent by the participants was logged. The third session also took place within the same week. After each test problem, mental effort was measured by asking the learner to fill out the 9-point rating scale. During the test phase participants were allowed to work at their own pace.

Results

The number of participants who reported having low, medium, and high prior knowledge on linear algebra was evenly distributed over the conditions ($X^2 = 1.98, p = .37$). Hence, the results reported here are not likely to be artefacts of prior knowledge differences.
between groups. Therefore, all subsequent analyses are performed using one-tailed independent samples t-test. For all statistical tests a significance level of .05 was maintained.

Training Phase

Table 3 provides an overview of the mean scores and standard deviations for the training and test variables.

[Insert Table 3 about here]

Performance. No effects on performance during the training session (sessions 1 and 2) were found. One of the experimenters, who was also the teacher who developed the problems for this study, divided the problems provided in three levels of complexity (low, medium, and high). The results of the perception test provided to students in the first test indicated that students prefer feedback on all the solution steps as the level of complexity of the problems increases. Therefore, additional analyses were carried out to investigate whether the effects on performance scores of the two types of feedback varied as a function of the complexity level of the problems. As Figure 4 depicts, for problems with a low and a medium complexity level, students performed better during training with feedback 'on the final solution'. For problems with a high complexity level, students performed better with feedback 'on all the solution steps'. However, the expected pattern found did not reach significance (all p’s > .05).

[Insert Figure 4 about here]

Mental effort. A t-test showed that the mental effort invested during training was higher for students in the feedback 'on the final solution' condition ($M = 4.44; SD = 1.18$) than for students in the feedback 'on all the solution steps’ condition ($M = 3.76, SD = .95; t(32) = 1.84, p < .05, d = .36$).

Time invested. No effects on training time between the experimental conditions were found.

Test Phase
Test Performance. A t-test on the near transfer test showed no significant differences between both feedback conditions. Analyses showed a significant difference between groups on far transfer performance, \( t(29) = -2.22; p < .015, d = -.80 \). Participants in the feedback ‘on all the solution steps’ condition scored higher on far transfer (\( M = 22.65, SD = 12.08 \)) than participants in the feedback ‘on the final solution’ condition (\( M = 13.65, SD = 10.48 \)).

Mental effort. No effects on invested mental effort during the test phase were found.

Efficiency. Analyses showed no significant differences on the efficiency of the near transfer test between the groups. A t-test revealed a significant difference on the efficiency of the far transfer test, \( t(25.72) = -1.86; p < .05 d = -.80 \). Efficiency scores were higher for participants in the feedback ‘on all the solution steps’ condition (\( M = .31, SD = .81 \)) than for participants in the feedback ‘on the final solution’ condition (\( M = -.36, SD = 1.23 \)).

Questionnaires

Table 4 provides an overview of the mean scores and standard deviations for the motivation and the perception questionnaires.

[Insert Table 4 about here]

Motivation questionnaire. A t-test showed a significant effect of feedback on relevance, \( t(32) = -2.75, p < .01, d = -.93 \); confidence, \( t(32) = -1.88, p < .05, d = -.63 \); and satisfaction, \( t(32) = -2.90, p < .001, d = -.99 \). Participants in the feedback ‘on all the solution steps’ condition reported higher relevance (\( M = 3.50; SD = .34 \)); higher confidence (\( M = 3.84; SD = .23 \)), and higher satisfaction (\( M = 3.43; SD = .36 \)) than participants in the feedback ‘on the final solution’ condition (\( M = 3.15; SD = .41; M = 3.56; SD = .59 \); and \( M = 3.06; SD = .39 \) for relevance, confidence, and satisfaction, respectively). A t-test showed no significant differences between the two groups on attention.
Perception questionnaire. A $t$-test showed a significant difference between groups on the participants’ perceptions of feedback, $t(31) = -8.83, p < .001$ $d = -3.02$. The mean in the perception scale of participants in the feedback 'on all the solution steps' condition ($M = 3.88$, $SD = .48$) was higher than the mean of participants in the feedback 'on the final solution' condition ($M = 2.16$, $SD = .66$). Similarly, a $t$-test showed a significant difference between participants’ perceived usefulness of feedback, $t(31) = -5.25, p < .0001$ $d = -1.84$. Perceived usefulness was significantly higher in the feedback 'on all the solution steps' condition ($M = 3.94$, $SD = .85$) than in the feedback 'on the final solution' condition ($M = 2.12$, $SD = 1.11$). In addition, Table 5 presents students’ responses on the “no / yes” questions. More participants in the feedback 'on the final solution' condition (82.4%) would like to see some features added to the feedback than in the feedback 'on all the solution steps' condition (29.4%; $X^2 = 9.66, p = .002$).

Conclusions

This study investigated the effects of two types of feedback, namely, feedback 'on the final solution' and feedback 'on all the solution steps' on learning effectiveness, efficiency, and motivation. We hypothesized that providing learners with feedback on all the solution steps would lead to higher effectiveness and efficiency. These hypotheses were largely supported by the findings of the present study. Learners provided with feedback on each solution step showed higher far transfer performance than learners provided with feedback on the final solution only. Learners provided with feedback on each solution step possibly constructed general cognitive schemas which enabled them to flexibly apply the learned solution procedure to solve unfamiliar linear algebra problems. Moreover, the higher efficiency on the far transfer test indicates that learners who received feedback on all the solution steps of the linear algebra problems not only show higher transfer test performance,
but also reach this performance with a proportionally lower investment of mental effort to reach that performance.

However, no differences between conditions were found for effectiveness and efficiency on the near transfer test. A possible reason could be that the general information available in the schemas constructed, although particularly useful to deal with tasks that require learners to *flexibly* apply the learned solution procedure, as in far transfer problems, it is of less use for familiar tasks that require learners to apply the learned solution procedure similarly to the practiced tasks, as in the near transfer problems (Kester, Kirschner, and van Merriënboer, 2006; Sweller et al., 1998).

In addition, the hypothesis that feedback on all the problem-solving steps will motivate the students more than feedback on the final solution was largely validated by the findings as well. Participants who received this form of feedback scored higher on three of the four scales of the IMMS (relevance, confidence, and satisfaction) than participants who were provided the more basic feedback. This could indicate that the feedback on each solution step indeed enabled students to connect what is presented to them to what they already know.

Furthermore, results support Keller’s (1983) theory of motivation, which argues that the motivation of a learner can be manipulated by the instructional design of the materials. In addition, students in the feedback ‘on all the solution steps’ condition perceived the feedback received as more positive and useful than students in the feedback ‘on the final solution step’ condition.

A final finding that needs to be discussed is the fact that mental effort invested during training was relatively low (all means are below the neutral score of 5). This could indicate that the problems were not too complex for the learners in general, and that the more elaborate type of feedback could have been useful in the most complex problems. Future studies may
examine the effects of elaborate and simpler forms of feedback on learners’ performance when more complex problems than the one used in this study are used.

General Conclusions

The aim of the two studies presented here was to investigate students’ perceptions (study 1) and the effects on learning and motivation (study 2) of feedback on all the problem-solving steps as compared to feedback ‘on the final solution step’ only. In both studies, participants were more positive about the feedback when this was provided on all the solution steps than about the feedback that was provided on the final solution step only. In addition, the second study also revealed advantages on the effectiveness, efficiency, and motivation of the feedback given on all the solution steps of the problem. More specifically, the second study has extended research on feedback by applying cognitive load theory by analyzing whether, in linear algebra problems, feedback on all the steps of the problem-solving process is more efficient than feedback on the final solution step only.

In addition, although in the second study the additional analyses on performance and level of problem difficulty during training did not yield significant differences, with regard to the most difficult problems the pattern was in the expected direction: Students in the feedback ‘on the final solution step’ condition scored lower ($M = 48.72, SD = 32.88$) than students in the feedback ‘on all the solution steps’ condition ($M = 61.18, SD = 26.34$). This is also in line with the students’ responses in study 1 in which they indicated to prefer step-by-step feedback (feedback on all the solution steps successively) and the worked examples (feedback on all the solution steps at once) for the most difficult linear algebra problem.

The means of the students’ perceptions of the feedback were rather similar in both studies. In general, students prefer feedback on all the solutions steps over feedback on the final solution step only. In both studies, participants filled out the questionnaire after receiving all the linear algebra problems linked to one specific feedback type. Future efforts might
examine whether students’ perceptions differ as a function of the difficulty level of the problems. This could be measured by asking students to fill out the questionnaire, or possibly just a single item, after each task or set of tasks from a certain difficulty level.

These findings and conclusions provide other implications for future research as well. First, it would be interesting to investigate whether gradually increasing the level of detail of the feedback along with the difficulty level of the problem yields better results on learning and motivation. In a feedback review (Mory, 2003) it is argued that providing learners simple but sufficient feedback in some situations may help minimizing the associated cognitive load to process the information because it might contain considerable less distracting information. In a recent study (Halabi, 2006), it was found that rich feedback was significantly more useful for students with no prior knowledge. Accordingly, future studies may dynamically adapt the feedback level to each individual learner’s expertise throughout the training. In addition, more advanced techniques such as eye-tracking or thinking-aloud protocols may uncover learners’ problem-solving processes. This might add valuable information to the learning measures used in the study and shed some light on how learners actually respond on the different feedback types. Finally, Kramarski and Zeichner (2001) found positive effects of metacognitive feedback (which used metacognitive questions that serves as cues for understanding) on performance in mathematical reasoning and explanations. Future studies might investigate the effects of metacognitive feedback on performance as well as on motivation.

To conclude, the two studies indicate that, for students of a technical domain, feedback provided on all the problem-solving steps is more effective, more efficient and it is clearly preferred and more motivating than feedback provided on the final solution step only. These results are particularly important because it shows the importance of the current efforts being made in developing learning tools that provide learners automatic generated feedback on the
problem-solving process as they progress throughout the training. More specifically, an important practical implication of this study for the design of electronic environments that offer automatically generated feedback in highly structured subjects, such as linear algebra, is that such electronic environments should incorporate step-wise guidance which supports students understand the correctness or incorrectness of their solutions.

References


Table 1

*Means and Standard Deviations of the Likert Scale Questions of the Feedback Specific Questionnaire (maximum score of 5)*

<table>
<thead>
<tr>
<th>Perception scale</th>
<th>Feedback type</th>
<th>On the final solution step</th>
<th>On all the solution steps at once</th>
<th>On all the solution steps successively</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$M$</td>
<td>$SD$</td>
<td>$M$</td>
</tr>
<tr>
<td>Perception scale</td>
<td></td>
<td>2.42</td>
<td>.92</td>
<td>3.94</td>
</tr>
<tr>
<td>Overall usefulness</td>
<td></td>
<td>2.44</td>
<td>1.23</td>
<td>4.00</td>
</tr>
</tbody>
</table>
Table 2

*Feedback Type Selected by the Participants*
### Table 3

**Means and Standard Deviations of the Training Phase and the Test Phase**

<table>
<thead>
<tr>
<th>Feedback type</th>
<th>On the final solution step</th>
<th>On all the solution steps at once</th>
<th>On all the solution steps successively</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Training Phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>61.15 (M), 21.80 (SD)</td>
<td>59.00 (M), 13.03 (SD)</td>
<td></td>
</tr>
<tr>
<td>Mental Effort</td>
<td>4.44 (M), 1.18 (SD)</td>
<td>3.76 (M), 0.95 (SD)</td>
<td></td>
</tr>
<tr>
<td><strong>Test Phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance on Near Transfer</td>
<td>32.65 (M), 10.517 (SD)</td>
<td>35.59 (M), 4.68 (SD)</td>
<td></td>
</tr>
<tr>
<td>Performance on Far Transfer</td>
<td>13.65 (M), 10.48 (SD)</td>
<td>22.05 (M), 12.08 (SD)</td>
<td></td>
</tr>
<tr>
<td>Mental Effort on Near Transfer</td>
<td>3.91 (M), 1.51 (SD)</td>
<td>3.38 (M), 1.08 (SD)</td>
<td></td>
</tr>
<tr>
<td>Mental Effort on Far Transfer</td>
<td>5.92 (M), 1.70 (SD)</td>
<td>5.68 (M), 1.59 (SD)</td>
<td></td>
</tr>
<tr>
<td>Efficiency on Near Transfer</td>
<td>-3.1 (M), 1.63 (SD)</td>
<td>26.05 (M), .59 (SD)</td>
<td></td>
</tr>
<tr>
<td>Efficiency on Far Transfer</td>
<td>-.36 (M), 1.23 (SD)</td>
<td>.31 (M), .81 (SD)</td>
<td></td>
</tr>
</tbody>
</table>

Set 1: Selected feedback type

1) Which of the three types of feedback provided do you prefer the **most**? 0 2 7
2) Which of the three types of feedback provided do you prefer the **least**? 7 0 2
3) Which of the three types of feedback was **easier** to follow? 0 4 5
4) Which of the three types of feedback was more **difficult** to follow? 6 0 3
5) Which of the three types of feedback was the most **informative**? 0 3 6
6) Which of the three types of feedback was the least **informative**? 6 0 3
7) Which of the three types of feedback was the most **informative**? 0 3 6
8) Which of the three types of feedback would you primarily choose for your learning? 0 3 6

Set 2: Given the problem below, which type of feedback would you choose?

1) Low difficulty 3 4 2
2) Medium difficulty 2 3 4
3) High difficulty 0 3 6
Means and Standard Deviations of the Questionnaires Provided after Session 2 (maximum score of 5)

<table>
<thead>
<tr>
<th>Feedback</th>
<th>On the final solution step</th>
<th>On all the Solution steps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Perception</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perception scale</td>
<td>2.16</td>
<td>.66</td>
</tr>
<tr>
<td>Overall usefulness</td>
<td>2.12</td>
<td>1.11</td>
</tr>
<tr>
<td>Motivation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attention</td>
<td>3.35</td>
<td>.30</td>
</tr>
<tr>
<td>Relevance</td>
<td>3.15</td>
<td>.41</td>
</tr>
<tr>
<td>Confidence</td>
<td>3.56</td>
<td>.59</td>
</tr>
<tr>
<td>Satisfaction</td>
<td>3.06</td>
<td>.39</td>
</tr>
</tbody>
</table>
Table 5

Response Percentages of the “No / Yes” Questions per Feedback Type in the Perception Questionnaire

<table>
<thead>
<tr>
<th>Feedback type</th>
<th>On the final solution step</th>
<th>On all the Solution steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Would you like to see some features added to this type of feedback?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% No</td>
<td>17.6</td>
<td>70.6</td>
</tr>
<tr>
<td>% Yes</td>
<td>82.4</td>
<td>29.4</td>
</tr>
<tr>
<td>Would you like to see some features removed from this type of feedback?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% No</td>
<td>94.1</td>
<td>76.5</td>
</tr>
<tr>
<td>% Yes</td>
<td>5.9</td>
<td>23.5</td>
</tr>
</tbody>
</table>
Figure Captions

*Figure 1.* Students’ selections of type of feedback as level of difficulty of exercises increases.

*Figure 2.* Screenshot of a training task presented to participants in the feedback ‘on the final solution only’ dealing with equations and basis (a) and of a training task presented to participants in the feedback ‘on all the solution steps’ dealing with matrices and transformations (b).

*Figure 3.* Example of a near transfer task (a) and of a far transfer task (b) (in Dutch).

*Figure 4.* Relationship between the two types of feedback and the level of complexity of the problems. *Note:* ‘Feedback 1’ means feedback ‘on the final solution step’, ‘feedback 2’ means feedback ‘on all the solution steps’.
Feedback in Linear Algebra Problems

Figure 1

Difficulty levels - Feedback selected

Level of difficulty

<table>
<thead>
<tr>
<th></th>
<th>Easy</th>
<th>Medium</th>
<th>Difficult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nr students</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

- Correct answer
- Worked out problem
- Step-by-step
Figure 2

The linear map $A : \mathbb{R}^3 \to \mathbb{R}^3$ satisfies

$A((3, -6, -9)) = (-30, -36, -72)$

$A((-3, 9, 36)) = (93, 126, 108)$

$A((2, -4, -3)) = (-14, -15, -7)$.

Find the matrix of $A$.

$$
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}

\begin{pmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{pmatrix}

\begin{pmatrix}
\frac{\epsilon}{\sqrt{1}} & \frac{\delta}{\sqrt{1}} \\
\frac{\epsilon}{\sqrt{1}} & \frac{\delta}{\sqrt{1}}
\end{pmatrix}

Check your answer

(a)

Determine an eigenvalue $\lambda$ and corresponding eigenvector $v$ for the matrix $A = 
\begin{pmatrix}
-13 & -4 \\
42 & 13
\end{pmatrix}$.

$$
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}

\begin{pmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{pmatrix}

\begin{pmatrix}
\frac{\epsilon}{\sqrt{1}} & \frac{\delta}{\sqrt{1}} \\
\frac{\epsilon}{\sqrt{1}} & \frac{\delta}{\sqrt{1}}
\end{pmatrix}

Check solution in steps

(b)
Find the inverse of the matrix:

\[
\begin{pmatrix}
1 & -2 & 2 \\
0 & 1 & 2 \\
1 & -2 & 3
\end{pmatrix}
\]

(a)

The plane \( V \) in \( \mathbb{R}^3 \) meets the kernel of the linear map \( A \) in a line through the origin. What is the dimension of \( A(V) \)?

(b)
Figure 4

Difficulty levels and Feedback

- Low
- Medium
- High